Vehicle Routing and Scheduling in a Supply Chain Network

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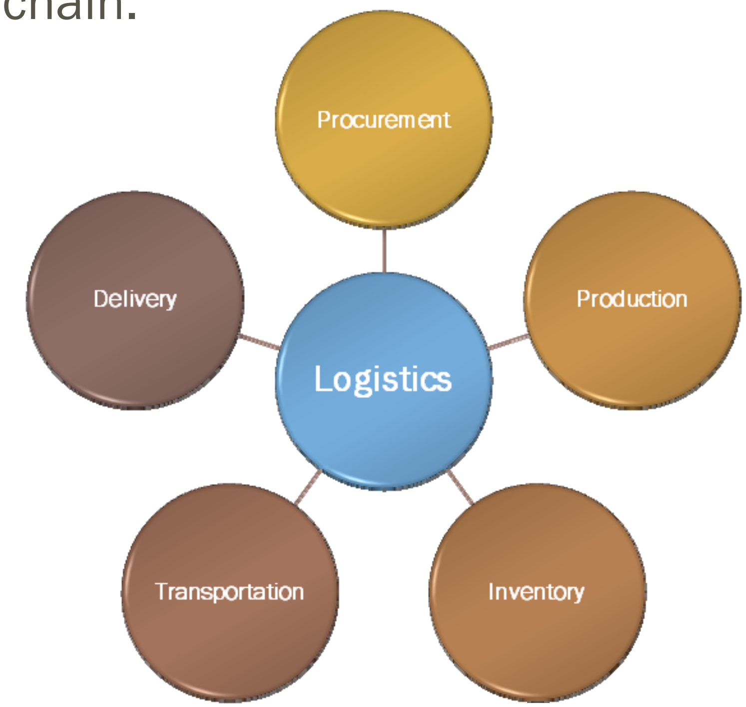
OUTLINE

- The distribution function
- Transportation modes and costs
- Distribution network design and planning
- Review of important routing and scheduling problems (TSP, VRP, PDP)
- The PDP with transsh ipment (PDPT)
- The VRP in multi-echelon networks with crossdocking
- Conclusions

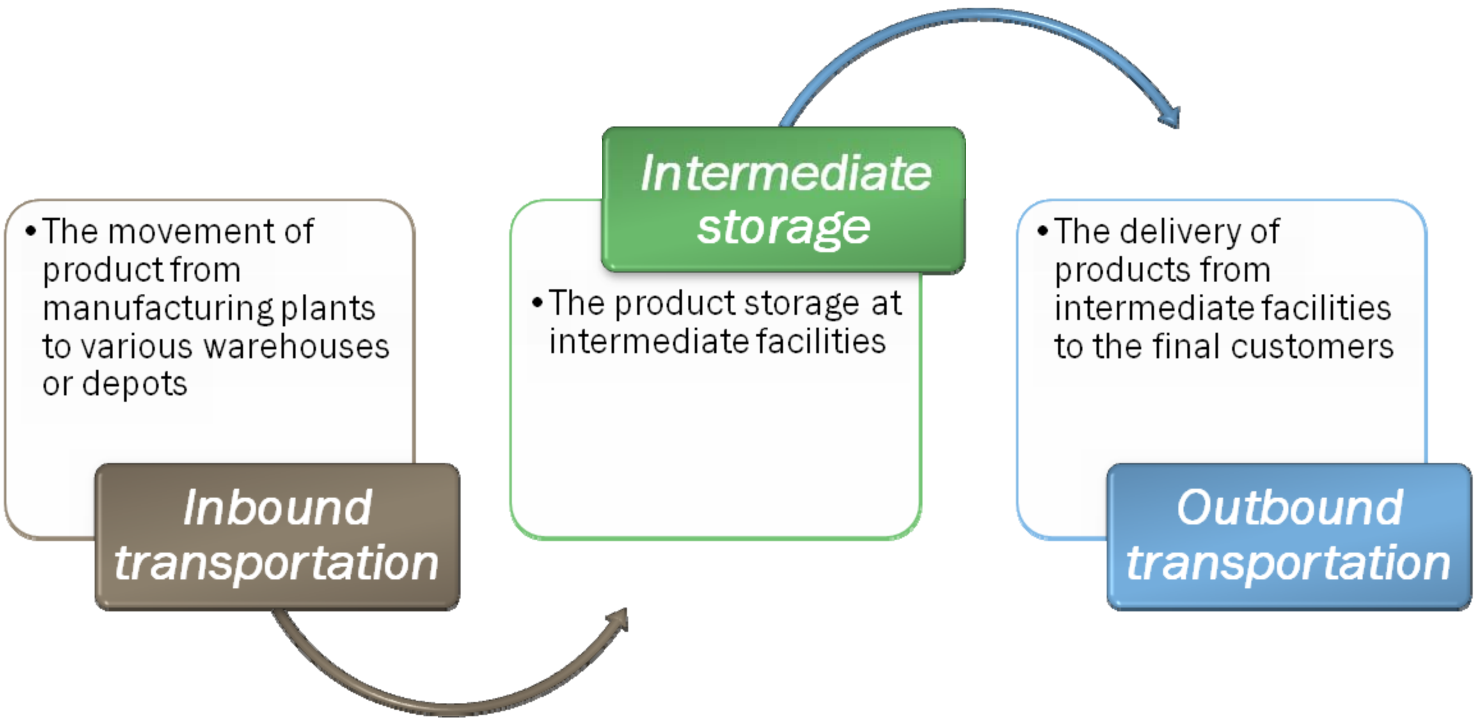
WHAT IS LOGISTICS ?

- **Logistics function:** provision of goods and services from supply points to demand points.
- It involves the management of a wide range of **business operations:** acquisition, production, storage, transportation and delivery of goods along the supply chain.



PHYSICAL DISTRIBUTION

- It comprises all movements and storage of goods "downstream" from the manufacturing plants.



THE ROLE OF TRANSPORTATION

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- Transportation play a key role because products are rarely produced and consumed at the same location.
- The last transportation step from distribution centers to customers (i.e. the outbound transportation), is usually the most costly link of the distribution chain.
- Distribution costs accounts for about 16% of the sale value of an item; approximately one fourth is due to the outbound transportation.

KEY PLAYERS IN TRANSPORTATION

- There are two key players in any transportation that takes place within a supply chain:

THE SHIPPER	THE CARRIER
✓It is the party requiring the movement of products between two points in the supply chain.	✓It is the party that moves the products.
✓He seeks to minimize the total cost, while providing an appropriate level of responsiveness to the customer.	✓He makes operating decisions trying to maximize the return from its assets

- For example, DELL uses UPS as the carrier to ship its computers from the factory to the customer.

TRANSPORTATION COSTS

- Types of transportation costs

Vehicle-related cost	It is the cost a carrier incurs for the purchase or lease of the vehicle used to transport goods.
Fixed operating cost	It includes the cost associated with terminals, airport gates that are incurred whether the vehicles are used or not.
Trip-related variable cost	This cost is incurred each time a vehicle leaves on a trip and includes the price of labour and fuel.
Quantity related cost	This category includes loading and unloading costs and a portion of the fuel cost that varies with the quantity being transported.
Overhead cost	It includes the cost of planning and scheduling a transportation network as well as any investment in information technology.

MODES OF TRANSPORTATION

- Listed by decreasing freight market share, the modes of transportation include:
  - *TRUCK*
    - TRUCKLOAD (TL)
    - LESS THAN TRUCKLOAD (LTL)
  - *WATER*
  - *RAIL*
  - *AIR*
  - *PIPELINE*

or a combination of them,

- *INTERMODAL TRANSPORTATION*

TRUCK

- Dominant mode of transportation in USA

- Two major segments:
  - full truckload (TL): charge for the full truck, and rates vary with the distance travelled.
  - less-than truckload (LTL): charge based on the quantity loaded and the distance travelled.

- Trucking is more expensive than rail but offers the advantage of :
  - door-to-door shipment
  - shorter delivery time
  - no transfer between pickup and delivery points

OTHERS MODES OF TRANSPORTATION

Rail

- High fixed cost in terms of rails, locomotives, cars, and yards
- Large, heavy, low-value shipments that are not very time-sensitive over long distances

Water

- Global Trade
- Very large loads
- Low costs
- Delays at ports and terminal

Air

- Very fast and fairly expensive
- Small, high-value items or time-sensitive emergency shipments

Pipeline

- Crude petroleum, refined petroleum products, and natural gas

- **Response Time (RT).** The time between the placement and the delivery of a customer order.
  - Location of warehouses closer to the market reduce RT.
  - *Trade-off between response time and inventory costs.* A decrease of RT produces an increase of both the number and cost of facilities, and the inventory costs.
  - Major issue to solve the trade-off: *Degree of inventory aggregation*

Transportation costs

- A higher number of warehouses lowers average distance to customers and outbound transportation costs.
- Warehousing allows consolidation of shipments from multiple suppliers in the same truck, to get lower inbound costs.
- Product customization at the delivery stage is postponed until receiving customer orders at the warehouse.
- *Major issues*: Consolidation of inbound shipments and temporal order aggregation (frequency of visits vs. full truckload) at the delivery stage.
- Trade-off between customer service level and outbound transportation costs.

PERFORMANCE MEASURES OF DISTRIBUTION NETWORKS

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- **Product Availability (PA).** It is the probability of having the requested product in stock when an order arrives.

  - **Direct shipping centralizes inventories** at the manufacturer site, and guarantees a high level of PA with lower amounts of inventories.
  - **Warehousing disaggregates inventories at intermediate facilities**, to lower response time and transportation costs, but decreasing the product availability.
  - For products with low/uncertain demand, or high-value products, all inventories are better aggregated at the manufacturer storage.
  - For low value, high-demand products, all inventories are better disaggregated and hold close to the customers.

DESIGN OF THE DISTRIBUTION NETWORK

- A well-designed transportation network allows a supply chain to achieve the desired degree of responsiveness at a low cost.

- According to the number of stocking levels, transportation network designs can be classified into two categories:

1. Single-echelon networks: Goods are directly shipped from suppliers to retail stores or customers.

2. Multi-echelon networks: Goods are shipped from suppliers to retail stores via intermediate stocking points

SINGLE-ECHELON TRANSPORTATION NETWORKS

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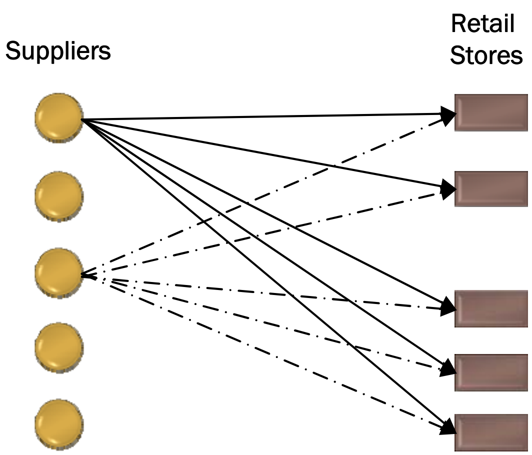
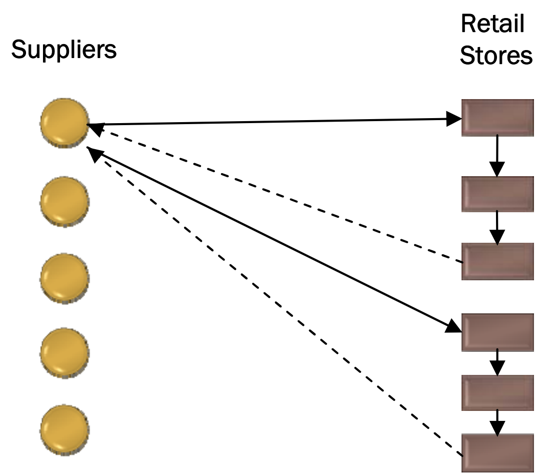


Direct shipping network

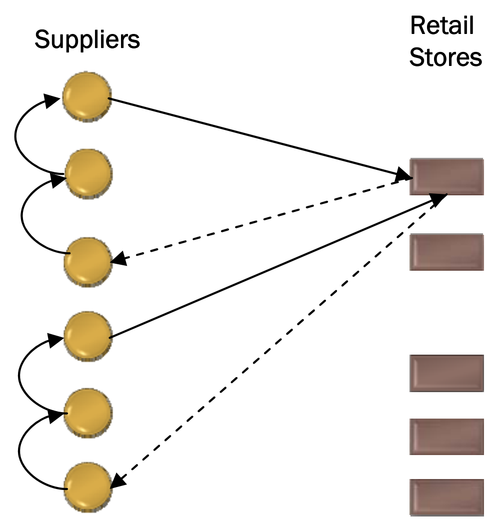
All shipments come directly from suppliers to retail stores



Direct shipping with milk runs (A)

A truck converging products from a supplier to multiple retail stores

Consolidation of shipments from a supplier to different destinations



Direct shipping with milk runs (B)

A truck converging products to a retail store from multiple suppliers

Consolidation of shipments to a retail store from different suppliers

MULTI-ECHELON TRANSPORTATION NETWORKS

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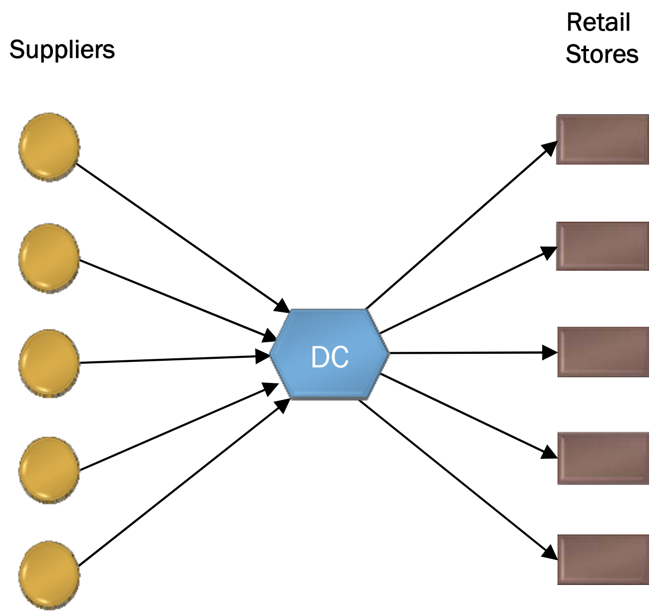
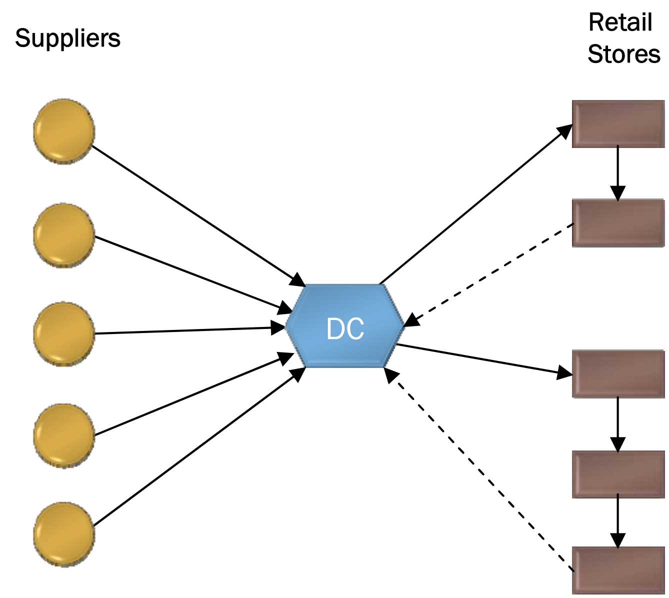


All shipments via a central DC



All shipments via distribution center using milk runs

OPERATIONS IN MULTI-ECHELON NETWORKS

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- **Consolidation** operations combine shipments from different suppliers and destined for multiple customers in the same truck.

- **Break-bulk** operations split a large shipment from various origins into multiple, smaller shipments.

- **Cross-docking** operations consist of performing break-bulk operations over ingoing, consolidated shipments right after their arrival at the intermediate facility, and immediately dispatching the customized parcels to their destinations.

DISTRIBUTION PLANNING LEVELS

- Distribution management involves a variety of decision-making problems at three levels: *strategic*, *tactical*, and *operational planning*.

- Strategic decisions deal with the distribution network design, including the number, location and size of facilities.

- Tactical decisions include: a) the area served by each depot; b) the fleet size and composition; c) inventory decisions at each facility; d) customer service levels.

- Operational decisions are concerned with the routing and scheduling of vehicles on a day-to-day basis.

VEHICLE ROUTING AND SCHEDULING PROBLEMS

TSP	Travelling salesman problem
MTSP	M-travelling salesman problem
VRP/VRPTW	Single depot, multi-vehicle, node routing problem without or with time windows
m-VRPTW	Multi-depot, multi-vehicle, node routing problem with time windows
PDPTW	Pickup and delivery problem with time windows
PDPT	Pickup and delivery problem with transhipment
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TRAVELLING SALESMAN PROBLEM (TSP)

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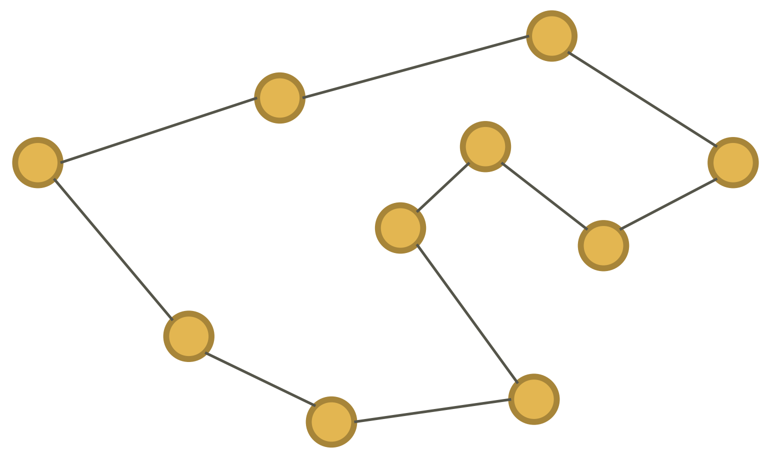
- Given a set of cities and the distances between them, determine the shortest path starting from a given city, passing through all the other cities and returning to the first town.

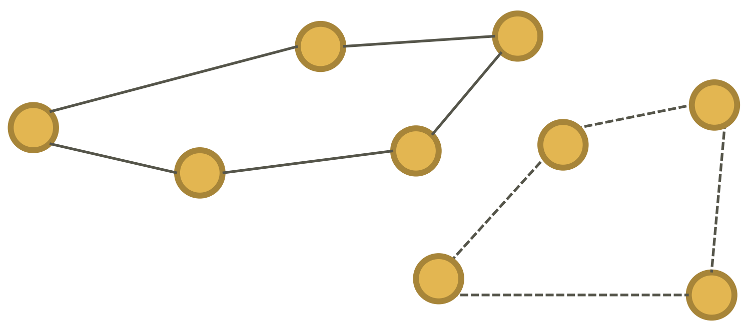
- It can be modeled as an undirected weighted graph, at which cities are the vertices, and paths are the edges.



MULTIPLE TRAVELING SALESMAN PROBLEM (m-TSP)

- It is a generalization of the travelling salesman problem where there is a need to account for more than one salesman.

- Given a set of cities and a set of salesmen, find the set of routes for the salesmen with a minimum total length so that:
  a) each salesman travels to a unique set of cities and completes the route by returning to the starting city
  b) each city is visited by exactly one salesman.



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1. Model-based exact approaches

- There is a single incident arc to each node.
- There is a single leaving arc from each node.
- Sub-tour elimination constraints guarantee that the cycle passes through all the cities and ends at the starting city.
- **The problem variables x_{ij} indicate whether an arc connecting nodes i and j is or is not in the selected tour (direct predecessor).**

TSP OPTIMAL APPROACH

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Variables

$$x_{ij} = \begin{cases} 1, & if\ arc(i,j) is\ in\ the\ tour \\ 0, & otherwise \end{cases}$$

c_{ij} = cost of the route (i, j) → $c_{ij} = c_{ji}$; $c_{ii} = +\infty$

Constraints

(R1) Each node has exactly a single incident arc

$$\sum_{i=1}^{n} x_{ij} = 1, \qquad j = 1,2,..,n$$

(R2) Each node has exactly a single leaving arc

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1,2,..,n$$

(R3)

$$x_{ij} = 0,1 \ ; \ x = (x_{ij}) \in S$$

(Objective function)

$$Min\ Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

→

The set *S* can be any restrictions that prohibit sub-tour solutions satisfying the assignment constraints (subtour-breaking constraints)

$$\sum_{i \in Q} \sum_{j \notin Q} x_{ij} \geq 1, for\ any\ subset\ Q\ of\ N$$

$$\sum_{j=1}^{n} y_{ij} - \sum_{j=1}^{n} y_{ji} = -1 \quad (i = 2,..,n)$$

$$y_{ij} \leq U x_{ij}$$

$$y_{ij} \geq 0$$

m-TSP FORMULATION

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- (OF) $$\text{Min } z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

subject to:

(R1) $$\sum_{i=1}^{n} x_{ij} = b_j = \begin{cases} M, \text{if } j = 1 \\ 1, \text{if } j = 2,3,\ldots,n \end{cases}$$

(R2) $$\sum_{j=1}^{n} x_{ij} = a_i = \begin{cases} M, \text{if } i = 1 \\ 1, \text{if } i = 2,3,\ldots,n \end{cases}$$

(R3) $$X = (x_y) \epsilon\, S$$

(R4) $$x_{ij} = 0,1 \;;\; i,j = 1 \ldots n$$

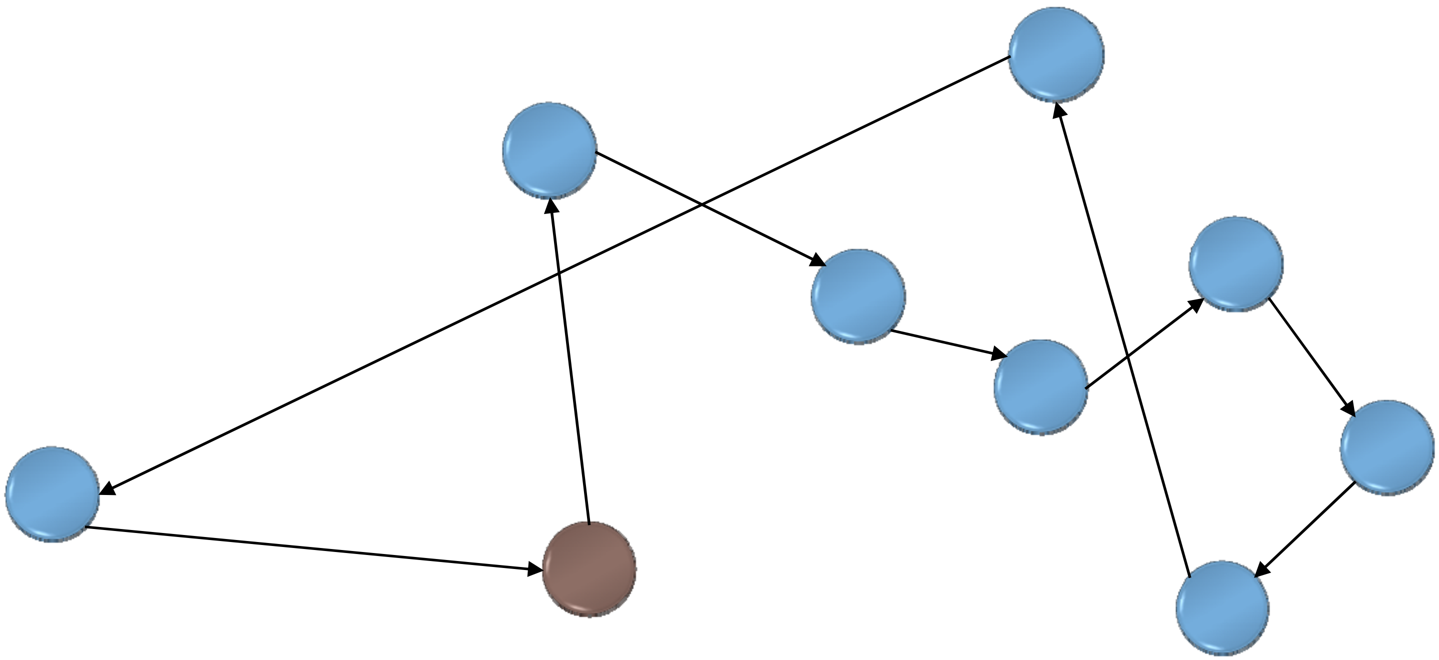
HEURISTIC APPROACHES FOR THE TSP

1. **Tour construction procedures.** Generate a feasible tour from the distance and saving matrices.

2. **Tour improvement procedures.** Find a better tour assuming that an initial tour is given, and perform arc/node exchanges between routes.

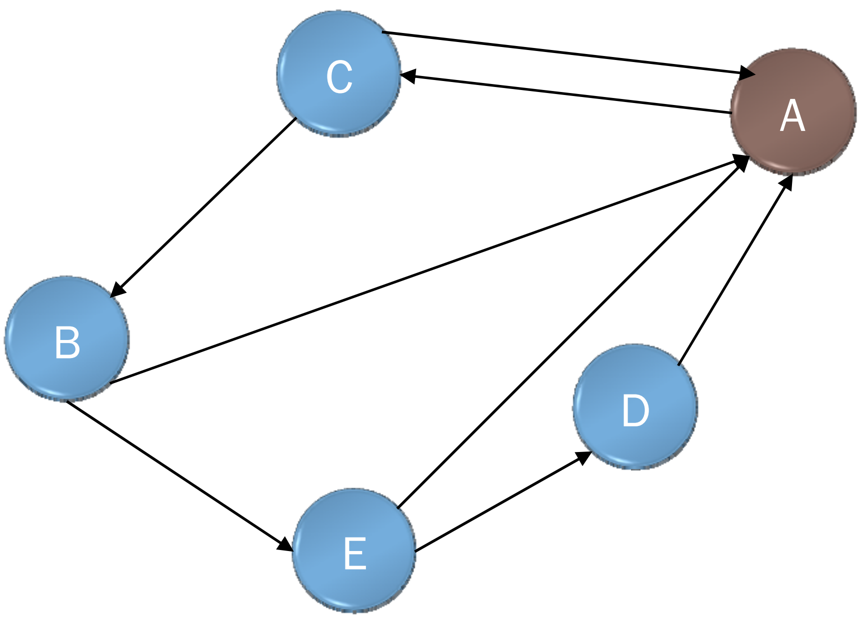
3. **Composite procedures.** Construct a starting tour using a tour construction procedure and find a better tour using a tour improvement procedure.

- Nearest neighbour



- Nearest insertion

	A	B	C	D	E
A	0	85	47	57	87
B	85	0	43	52	38
C	47	43	0	48	58
D	57	52	48	0	32
E	87	38	58	32	0



TOUR IMPROVEMENT PROCEDURES FOR THE TSP

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2-OPT procedure

- Break the initial tour at two points.
- The resulting 2 paths can be reconnected in two ways.
- Choose the shortest one as the new tour.

3-OPT procedure

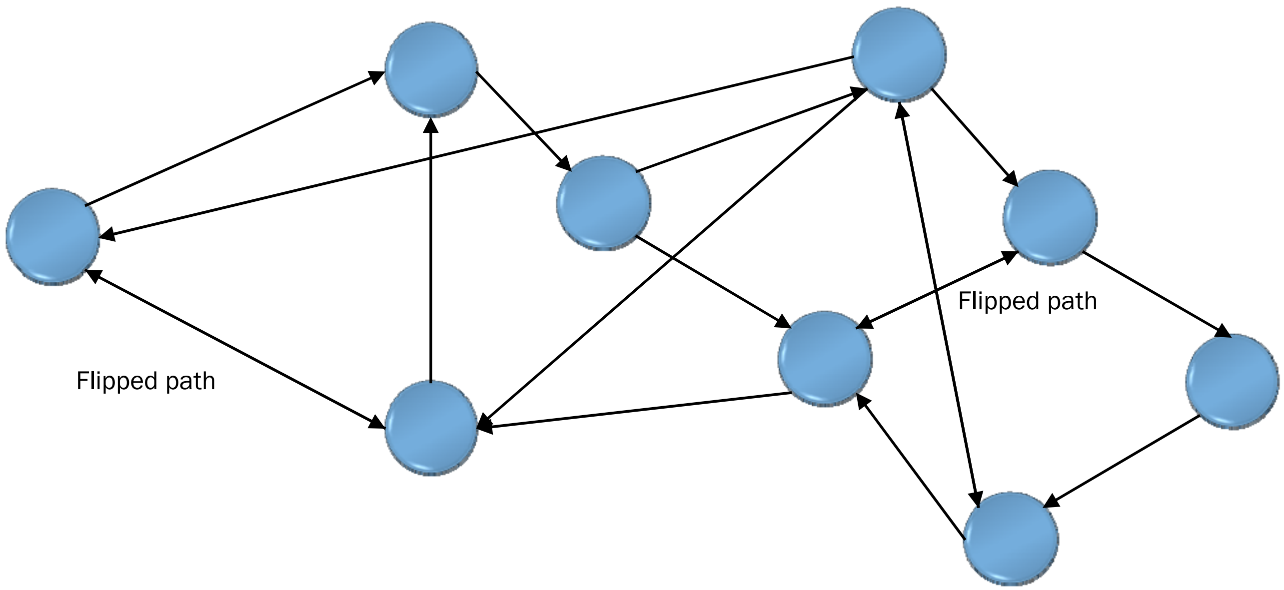
- Break the initial tour at three points.
- The resulting 3 paths can be reconnected in 8 ways.
- Choose the shortest one as the new tour.

k-OPT procedure

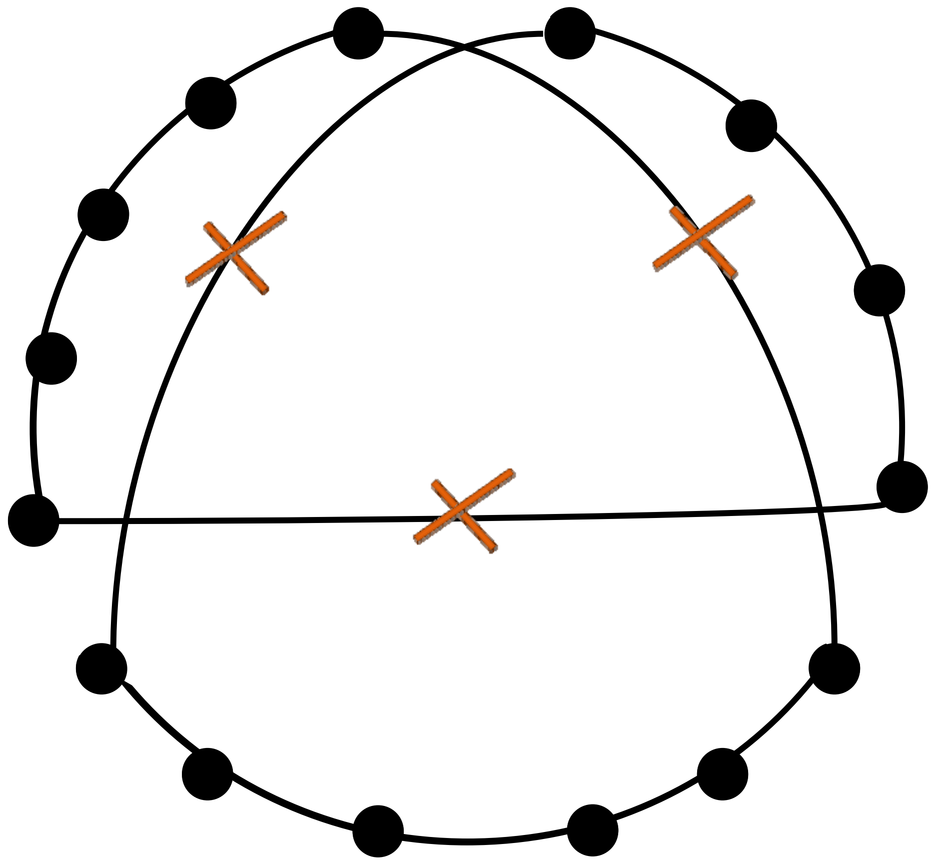
- Break the initial tour at k points.

TOUR IMPROVEMENT PROCEDURES FOR THE TSP

- 2-OPT procedure



- 3-OPT procedure



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VEHICLE ROUTING PROBLEM (VRP)

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- Generalization of the m-TSP, where a **demand is associated** with each node and every **vehicle has a finite capacity**.

- In VRP, the sum of fixed costs (associated to the number of used vehicles) and variable costs (associated to the total distance traveled) is minimized.

GOAL

*Generate **optimal routes for the vehicle fleet** based on a given road network so as to **meet customers demands** while **satisfying capacity and time constraints at minimum transportation cost***

THE CLASSICAL VRP ASSUMPTIONS

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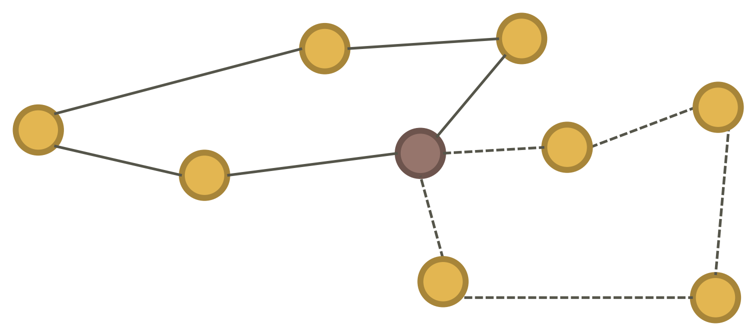
- The **demand** at each node is assumed to be **deterministic.**
- Each vehicle has a **known capacity** that cannot be exceeded.
- Each vehicle route **must start and end at the central depot.**
- Each node **must be visited by exactly one vehicle.**



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1. Routing Problem
   - **A spatial problem. Temporal considerations are ignored.** No a priori restrictions on delivery times (i.e. no TW constraints) and goods can be delivered within a short period of time (i.e. non-active maximum service time constraints).

2. Routing and Scheduling Problem
   - **Visiting times to various locations are of primary importance**. Temporal considerations may no longer be ignored and time restrictions guide the routing and scheduling activities. The movement of each vehicle must be traced through both time and space.

SOLUTION TECHNIQUES FOR VRP PROBLEMS

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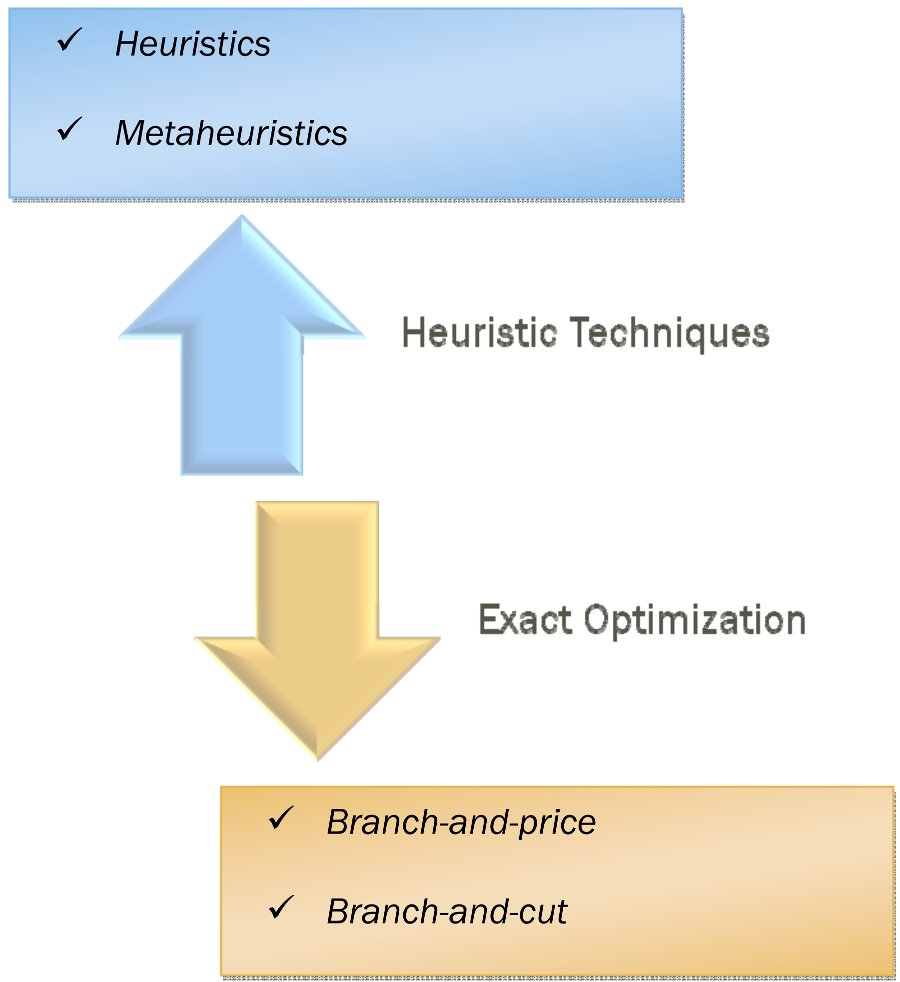
Heuristic methods

a) Tour-construction procedures
b) Tour-improvement procedures
c) Composite procedures

Metaheuristic techniques



Exact Approaches

TOUR CONSTRUCTION PROCEDURES

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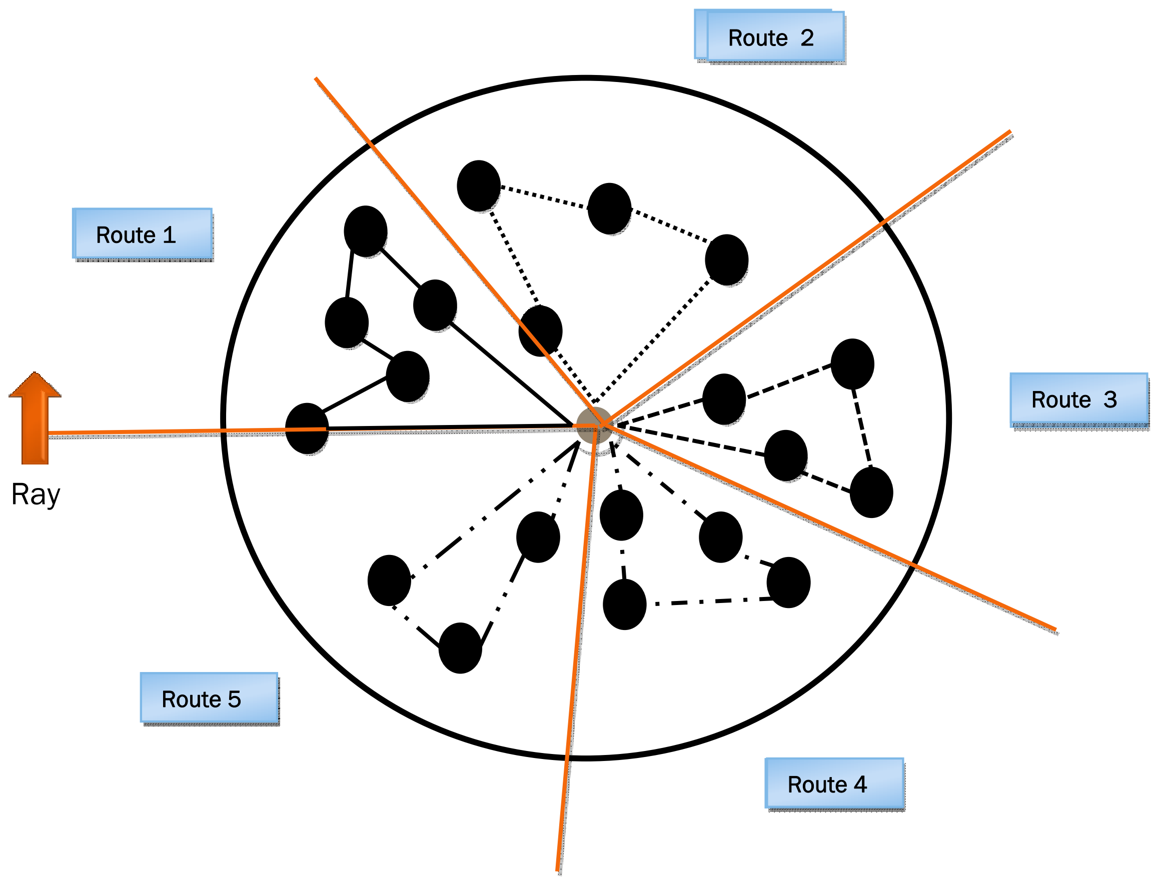
1. **Cluster first- route second procedures.** Group first demand nodes into clusters and then design economical routes over each cluster.

2. **Route first-cluster second procedures.** A long route or cycle that includes all the demand nodes is constructed. Next, the long route is partitioned into a number of shorter, feasible routes by defining clusters.

3. **Saving/insertion procedures.** Build a solution by moving from the current one to an alternative configuration that yields the largest savings in terms of some criterion function like the total cost.

4. **Interactive optimization.** It is a general-purpose approach in which a high degree of human interaction is incorporated into the problem-solving process.

- Cluster First – Route Second



TOUR IMPROVEMENT PROCEDURES

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- **k-string exchange**. Exchange of two strings with at most k nodes within the same route or between neighboring routes.
- **k-node exchange**. Exchange of a certain number of nodes between routes.
- **k-string relocation**. Moving a string of at most k nodes (with k =1-2) from one to another route.
- **k-OPT procedure**. It randomly breaks a trip at k points into k paths and reconnecting them in all possible ways.
- **string cross**. Exchange of two node-strings on two routes by crossing two edges of such routes.
- **k-cyclic transfer**. Given routes (r1, r2, r3), move k nodes from r1 to r2, a similar number from r2 to r3 and, finally, k customers from r3 to route r1.

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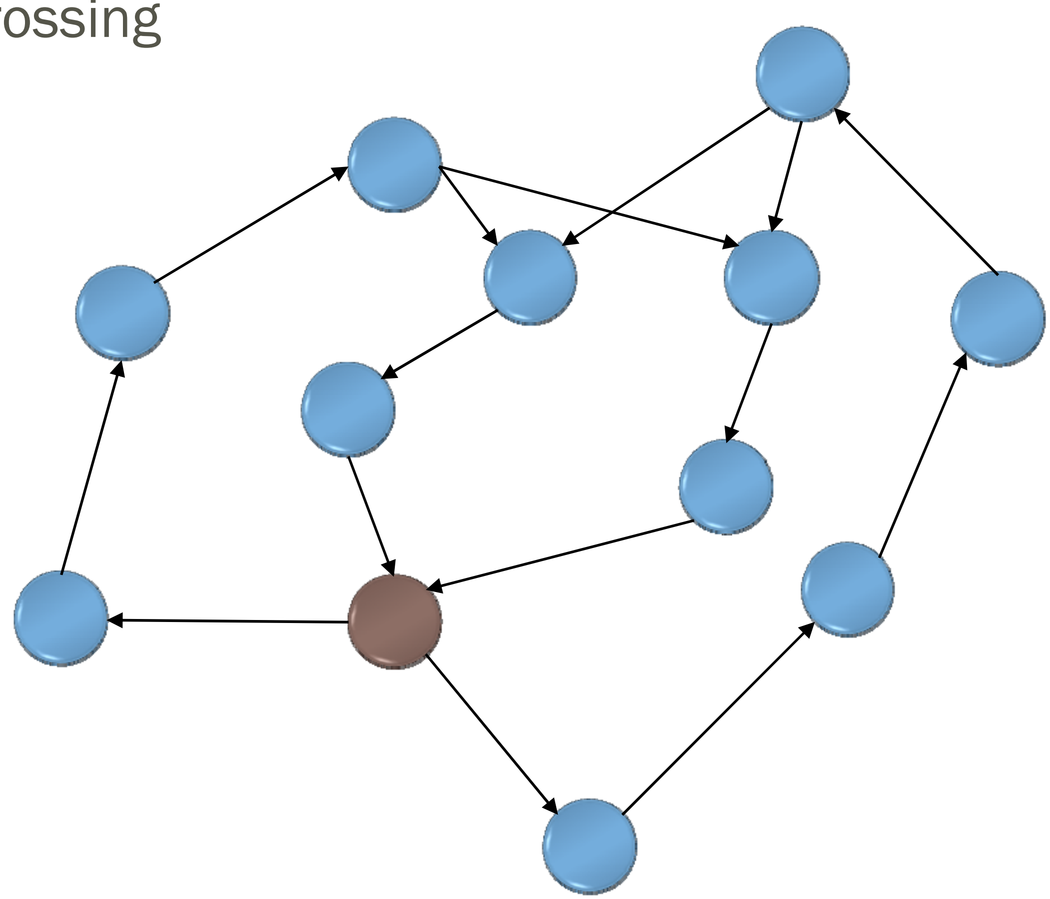
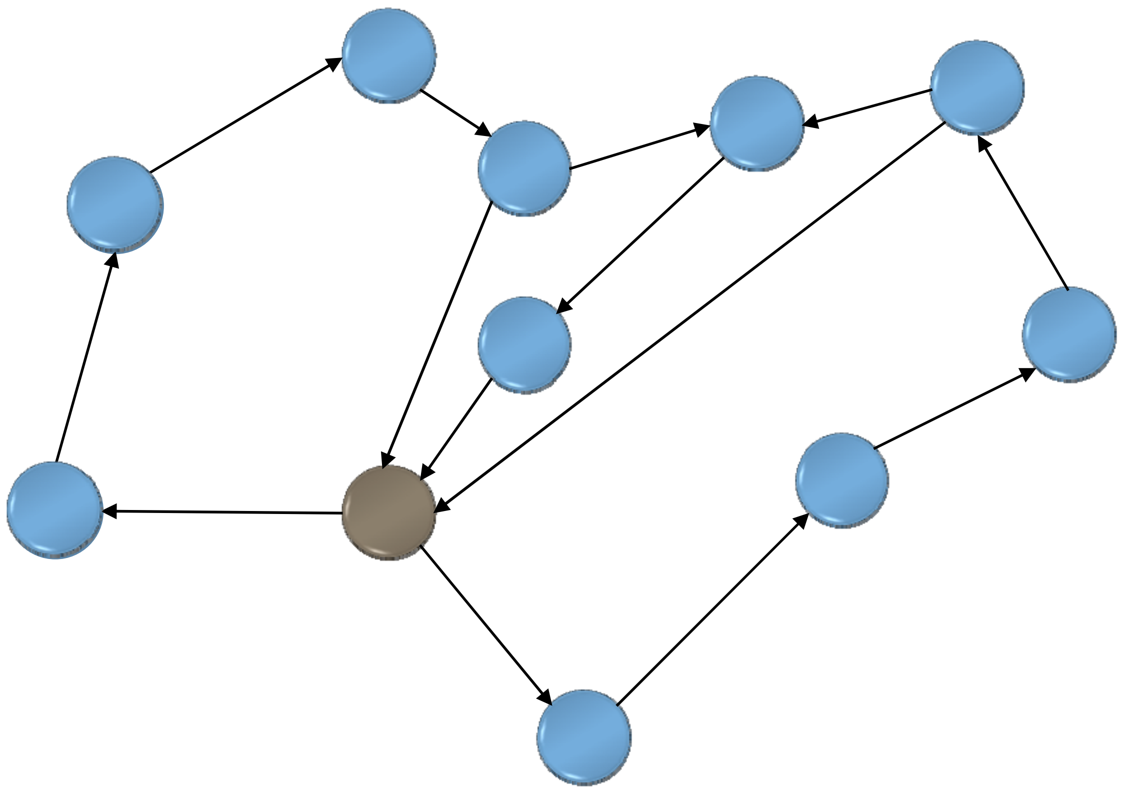
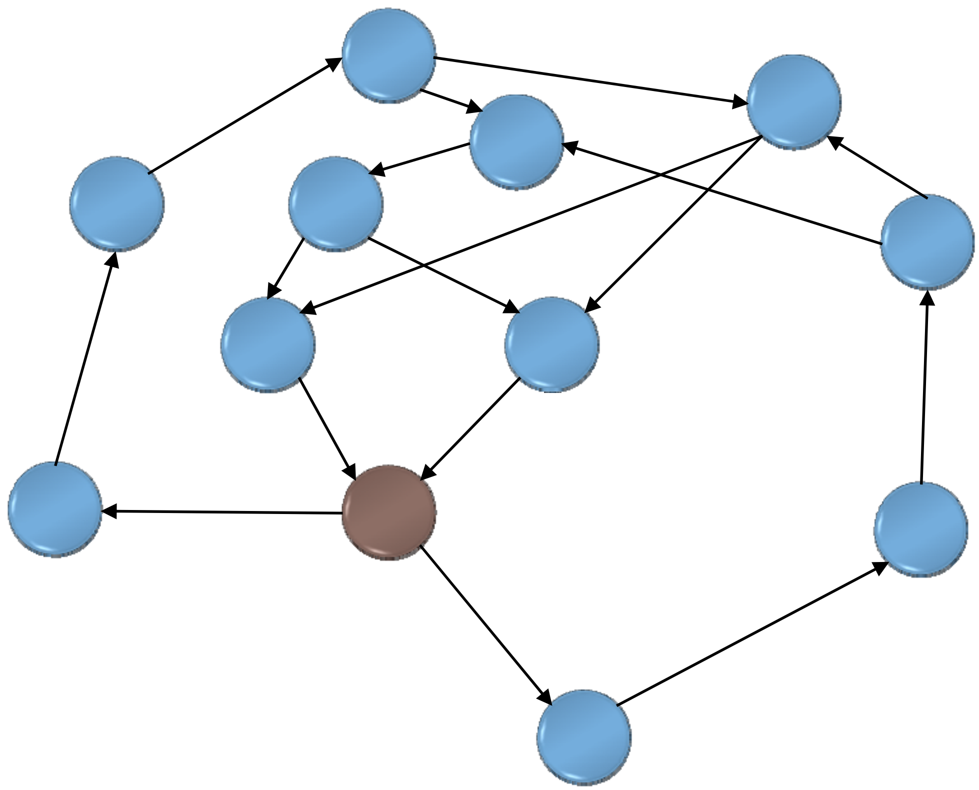
- String Crossing



- String Relocation



- String Exchange



COMPOSITE PROCEDURES

- Apply first a tour construction procedure and then a tour improvement procedure.

1. Savings matrix method: Use the cost savings matrix to generate the initial routes.

2. Generalized assignment method: Use the notion of seed point of a tour to generate the initial routes.

METAHEURISTIC TECHNIQUES FOR THE VRP

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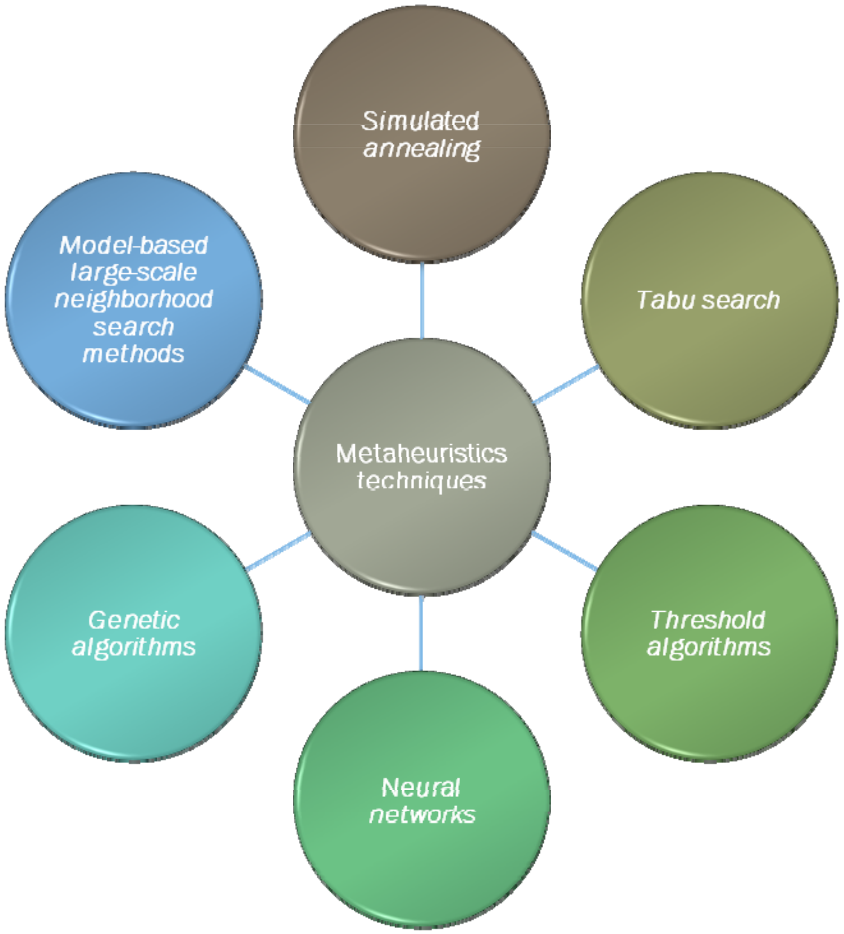
- Metaheuristic is an iterative process driven by some subordinate heuristic.

- They are mostly tour improvement procedures. Start with a non-optimal set of feasible tours and seek out a better solution through local perturbations.

- Iteratively use some version of a local search method to obtain a new set of lower-cost, feasible vehicle routes.

- A critical issue is the choice of the neighborhood structure.

METAHEURISTIC TECHNIQUES FOR THE VRP



- Explore a set of neighbors obtained from the current solution by doing a *limited number of moves*.

- The larger the neighborhood,
  - the better is the quality of the best neighbor and the higher is the likelihood of converging to the truly optimal solution.
  - But the longer the time it takes to complete the search.

- A large neighborhood is not always the best option unless it is explored in a very efficient manner.

- Four parameters influence the computational behavior of local improvement procedures:
  - the initial solution.
  - the type of string moves and the string length *k* allowed
  - the improvement strategy used for choosing the next incumbent solution, i.e. the first-improved neighbor (FI) or the best neighbor (BI).

- Cross Point

Parent 1	1	2	3	4	5	6	7	8	9
Proto Child	1	2	3	4	--	--	--	--	--
Final Child	1	2	3	4	7	8	5	9	6
Parent 2	7	3	1	8	5	2	9	6	4

- Inversion Mutation

1	2	3	4	5	6	7	8	9

1	2	3	4	8	7	6	5	9

MODEL-BASED LARGE-SCALE NEIGHBORHOOD SEARCH METHOD

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- It steadily improves an initial solution by iteratively applying a sequence of two evolutionary steps: the route improvement and the route reoptimization steps.

- The neighborhood is defined through an MILP formulation that allows multiple nodal exchanges between neighboring trips. The best neighbor is the new incumbent solution (***the MILP improvement step***).

- Next, a new neighborhood and a new MILP formulation just allowing the repositioning of nodes on the same tour are defined (***the MILP route reoptimization step***).

- In both steps, the best neighbor is the one minimizing the overall routing cost, including fixed and variable costs.

- ***A perturbation mode*** is activated whenever no better neighbor is found through the normal procedure. It explores a larger neighborhood by allowing nodal exchanges among close trips and node reordering on every route.

EXACT APPROACHES FOR THE VRPTW

1. Optimization algorithms based on Lagrangian relaxation
2. Branch-and-price algorithms (column generation methods)
3. Branch-and-cut algorithms applied to MILP rigorous formulations of the VRPTW problem

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- They apply to MILP mathematical formulations and guarantee the optimality of the solution found. They are regarded as route construction methods.

Objective Function

$$Min \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{v=1}^{NV} c_{ij}x_{ij}{}^{v} + f_v \sum_{j=1}^{n} x_{ij}{}^{v}$$

(R1) A single leaving arc from each node

$$\sum_{i=1}^{n}\sum_{v=1}^{NV} x_{ij}{}^{v} = 1 \quad , \qquad j = 2,..,n$$

(R2) A single incident arc from each node

$$\sum_{j=1}^{n}\sum_{v=1}^{NV} x_{ij}{}^{v} = 1 \quad , \qquad i = 2,..,n$$

(R3) Each node including the depot has a similar number of incident and leaving arcs

$$\sum_{i=1}^{n} x_{ip}{}^{v} - \sum_{j=1}^{n} x_{pj}{}^{v} = 0\,, \qquad v = 1..NV, p = 1..n$$

(R4) Vehicle capacity constraints

$$\sum_{i=1}^{n} d_i \left(\sum_{j=1}^{n} x_{ij}{}^{v}\right) \le k_v\,, v = 1..NV$$

(R5) Total elapsed routing time constraint

$$\sum_{i=1}^{n} t_{iv} \sum_{j=1}^{n} x_{ij}{}^{v} + \sum_{i=1}^{n}\sum_{j=1}^{n} t_{ij}{}^{v} x_{ij}{}^{v} \le T_v$$

(R6) At most a leaving arc from the depot on each route

$$\sum_{J=2}^{n} x_{ij}{}^{v} \le 1\,, v = 1..NV$$

(R7) At most an incident arc to the depot on each route

$$\sum_{i=2}^{n} x_{ij}{}^{v} \le 1\,, v = 1..NV$$

(R8) Subtour breaking constraints

$$x \in S$$

$$x_{ij}{}^{v} = 0,1$$

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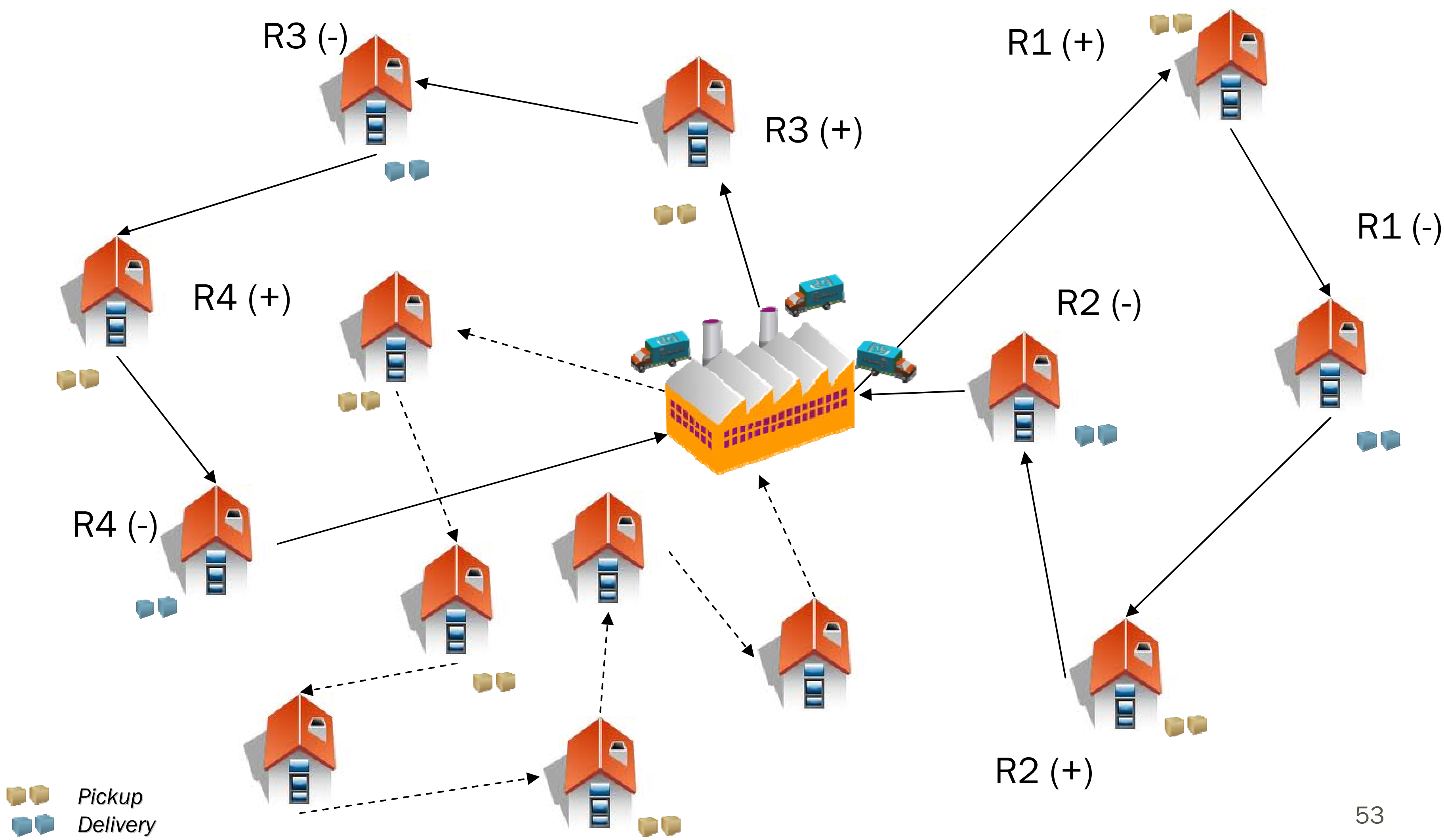
PICKUP & DELIVERY PROBLEM (PDP)

- A generalization of the VRP where ***pickup and delivery operations*** are done by a fleet of vehicles.
- It is the multi-vehicle pickup and delivery problem with time-windows (m-PDPTW).
- It is a single-echelon transportation problem involving pickup and delivery tasks, but accomplishing a single type of operation at each node.
- Involve a set of transportation requests $r \in R$, each defined by:
  - a pickup location (the origin),
  - a delivery location (the destination),
  - a load to be delivered from one to the other site, and
  - a service time at each location.
- All the requests are known in advance.

m-PDPTW



m-PDPTW PROBLEM FEATURES

- Vehicles depart and return to the central depot (tour constraint).
- Each transportation request must be serviced by a single vehicle. Related pickup and delivery locations are then visited by the same vehicle (pairing constraint).
- Each pickup location has to be visited prior to the corresponding delivery location (precedence constraint).
- Each vehicle can satisfy one or more customer requests.

m-PDPTW PROBLEM FEATURES

- A vehicle capacity must never be exceeded after visiting a pickup node (capacity constraint at pickup nodes).

- A vehicle must transport enough load to meet customer demand when servicing a delivery node (capacity constraint at delivery nodes).

- The service at each node must be started within the specified time window (time window constraints).

- The total time/distance travelled from the depot to a certain node must be greater than the one required to reach a preceding node on the tour (compatibility between routes and schedules).

m-PDPTW PROBLEM FEATURES

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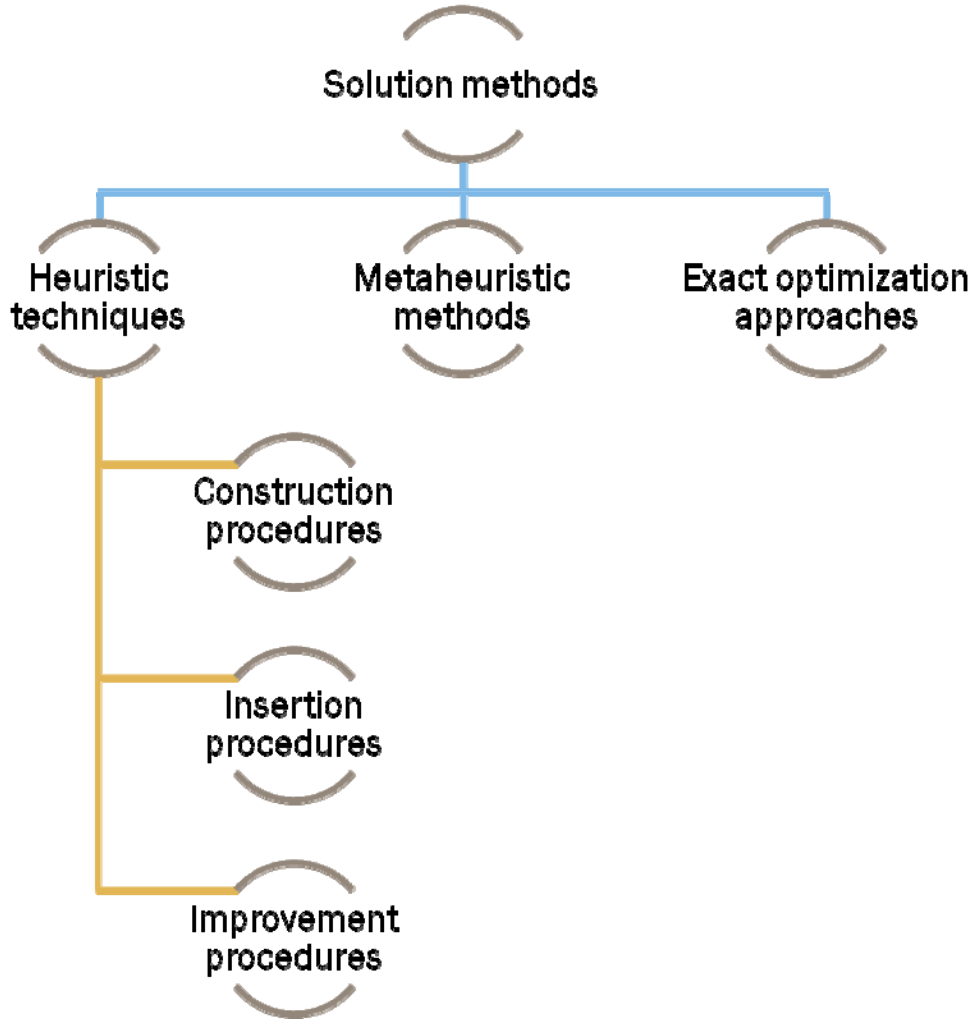
- The problem goal is to minimize:
  - the total distance
  - the total time required to service all customer requests
  - the total customers' inconvenience
  - a weighted combination of total service time and customers' inconvenience.

- Customer inconvenience is usually a linear function of a customer's waiting time (time window violation).

m-PDPTW SOLUTION METHODS



CONSTRUCTION HEURISTIC METHODS

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- The best construction heuristic methods are decomposition techniques based on the idea of dividing the problem into three phases:
  - clustering
  - routing
  - scheduling

- Decomposition is based on the notion of *mini-clustering*.

- Routing and scheduling are accomplished using methodologies proposed for the VRP subject to the pairing and precedence constraints.

INSERTION HEURISTIC METHODS

- These methods develop a set of routes by inserting one request at a time into a given route.

- Two types of insertion procedures have been proposed:

  - Sequential procedures inserting one customer at a time into a single route.
  - Parallel procedures inserting one customer at a time into one of several open routes.

IMPROVEMENT HEURISTIC METHODS

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- Start from a set of feasible routes and applied some kind of *local search algorithm* to improve the current solution.

- The neighborhood structure depends on the procedure.

- Cyclic transfer procedure improved the solution by moving a number of requests among routes.

- Variable depth search applied several arc-exchange mechanisms to improve the current solution.

METAHEURISTIC ALGORITHMS FOR PDP

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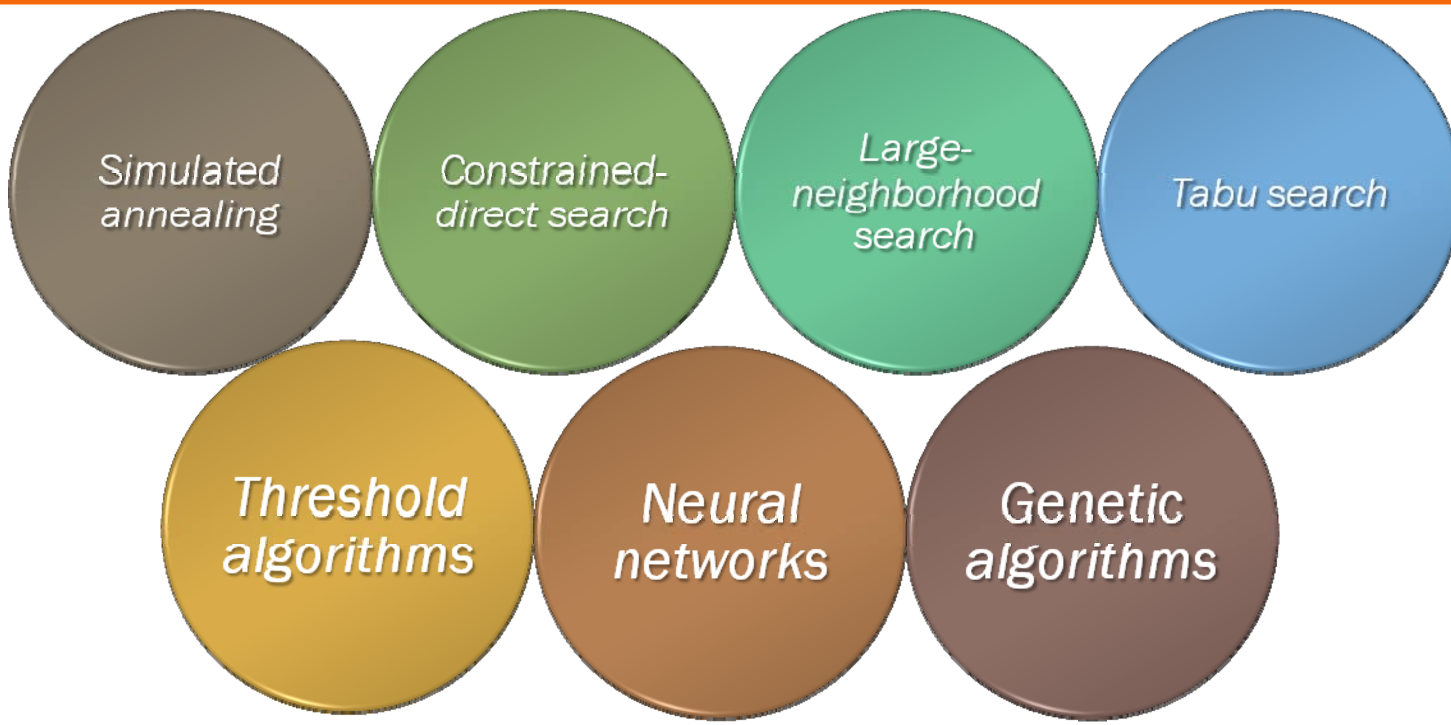

- Most of these methods seek to escape from a local optima by:
  - Restarting the search procedure from the current best solution after several non-improving iterations.
  - Redefining the solution space and doing re-optimization to continually improve on the current best solution.

m-PDPTW FORMULATION

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ASSIGNMENT CONSTRAINTS

$$\sum_{p\in P} X_{vp} \le 1 \quad ,\forall v\in V$$

$$\sum_{v\in V} Y_{rv} = 1 \quad ,\forall r\in R$$

$$\sum_{r\in R} Y_{rv} \le M_X \sum_{p\in P} X_{vp} \qquad \forall v\in V$$

ROUTING-COST DEFINING CONSTRAINTS

$$C_i \ge C_{pi}\left(X_{vp} + Y_{rv} - 1\right), \qquad \forall i\in I^+{}_r, r\in R, v\in V, p\in P$$

$$C_{i'} \ge C_i + c_{ii'} - M_C\left(1-S_{ii'}\right) - M_C\left(2-Y_{rv}-Y_{r'v}\right)$$

$$C_i \ge C_{i'} + c_{ii'} - M_C S_{ii'} - M_C\left(2-Y_{rv}-Y_{r'v}\right) \qquad \forall i\in I_r, i'\in I_r, i<i', (r,r')\in R, v\in V$$

$$OC_v \ge C_i + \sum_{p\in P} c_{ip} X_{vp} - M_C\left(1-Y_{rv}\right) \qquad \forall i\in I_r{}^-, r\in R, v\in V$$

VEHICLE-LOAD DEFINING CONSTRAINTS

$$L_i - U_i \le \sum_{v\in V_r} q_v Y_{rv}$$

$$L_i - U_i \ge 0 \qquad \forall i\in I_r, r\in R$$

$$L_{i'} \ge L_i + \alpha_{i'} - M_T\left(1-S_{ii'}\right) - M_T\left(2-Y_{rv}-Y_{r'v}\right)$$

$$L_i \ge L_{i'} + \alpha_i - M_C S_{ii'} - M_C\left(2-Y_{rv}-Y_{r'v}\right) \qquad \forall i\in I_r, i'\in I_r, i<i', (r,r')\in R, v\in V$$

$$U_{i'} \ge U_i + \beta_{i'} - M_T\left(1-S_{ii'}\right) - M_T\left(2-Y_{rv}-Y_{r'v}\right)$$

$$U_i \ge U_{i'} + \alpha_i - M_C S_{ii'} - M_C\left(2-Y_{rv}-Y_{r'v}\right) \qquad \forall i\in I_r, i'\in I_r, i<i', (r,r')\in R, v\in V$$

$$L_i \le \sum_{r'\in R} q_v{}^0 Y_{r'v} + \sum_{r'\in R}\sum_{j\in I_r} \alpha_j Y_{r'v} - M_L\left(1-Y_{rv}\right) \qquad \forall i\in I_r, r\in R$$

$$U_i \le \sum_{r'\in R}\sum_{j\in I_r} \beta_j Y_{r'v} - M_L\left(1-Y_{rv}\right) \qquad \forall i\in I_r, r\in R$$

$$U_i \ge \beta_i$$

$$L_i \ge \sum_{r\in R} q_v^0 Y_{rv} + \alpha_i \qquad \forall i\in I_r, r\in R$$

ARRIVAL-TIME DEFINING CONSTRAINTS

$$T_i \ge T_{pi}\left(X_{vp} + Y_{rv} - 1\right), \qquad \forall i\in I^+{}_r, r\in R, v\in V, p\in P$$

$$T_{i'} \ge T_i + st_i + c_{ii'} - M_T\left(1-S_{ii'}\right) - M_T\left(2-Y_{rv}-Y_{r'v}\right)$$

$$T_i \ge C_{i'} + st_{i'} + c_{ii'} - M_C S_{ii'} - M_C\left(2-Y_{rv}-Y_{r'v}\right) \qquad \forall i\in I_r, i'\in I_r, i<i', (r,r')\in R, v\in V$$

$$T_j \ge T_i + st_i + t_{ij} \qquad (i,j)\in I_r, i\in I^+, j\in I^-, r\in R$$

$$OT_v \ge T_i + st_i + \sum_{p\in P} c_{ip} X_{vp} - M_C\left(1-Y_{rv}\right) \qquad \forall i\in I_r{}^-, r\in R, v\in V$$

$$E_i \ge a_i - T_i \qquad \forall i\in I$$

$$B_i \ge T_i - b_i \qquad \forall i\in I$$

$$OD_v \ge OT_v - t_v{}^{\max} \qquad \forall v\in V$$

OBJECTIVE FUNCTION

$$Min \sum_{v\in V}\left(OC_v + \Gamma OT_v + \Lambda OD_v\right) + \sum_{i\in I}\left(\lambda E_i + \omega B_i\right)$$

VEHICLE ROUTING AND SCHEDULING PROBLEMS

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TSP	Travelling salesman problem
MTSP	M-travelling salesman problem
VRP/VRPTW	Single depot, multi-vehicle, node routing problem without or with time windows
m-VRPTW	Multi-depot, multi-vehicle, node routing problem with time windows
PDPTW	Pickup and delivery problem with time windows
PDPTWT	Pickup and delivery problem with transhipment
VRP-SCM	Vehicle routing and scheduling problem in a multi-echelon supply chain
VRPCD-SCM	Vehicle routing and scheduling problem with crossdocking in a multi-echelon supply chain

PDPTW WITH TRANSHIPMENT

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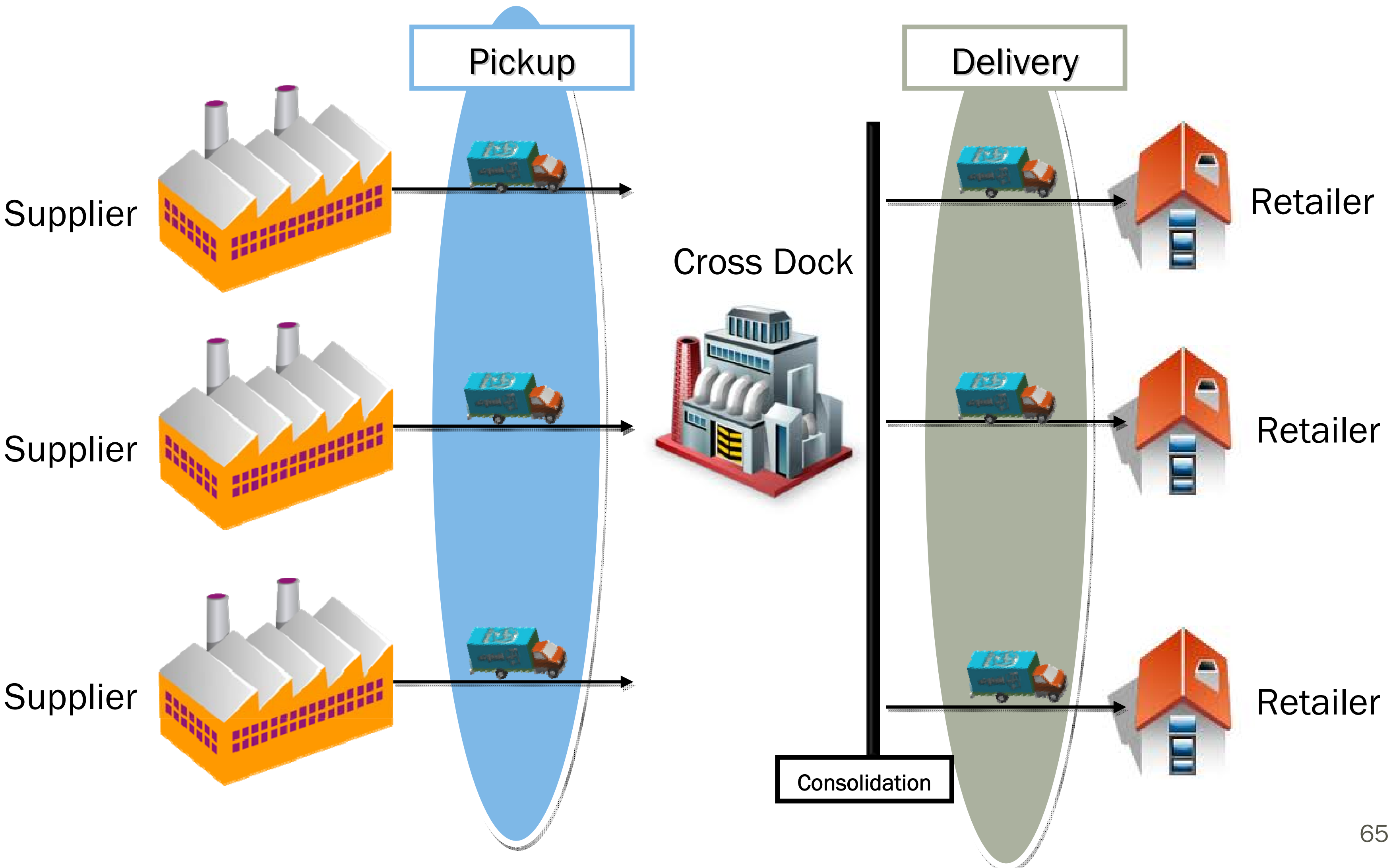
- Loads are transported from suppliers (pickup nodes) to customers (delivery nodes) via a single cross-dock.

- Each request is split into two sub-requests, i.e. a pickup and a delivery sub-request that may be handled by two different vehicles.

- The incorporation of transshipment points may yield solutions with shorter travel distances or fewer vehicles.

- Loads from suppliers are picked up by a fleet of vehicles, consolidated at the cross-dock facility, and immediately delivered to customers by the same set of vehicles, without intermediate storage.

PDPTW WITH TRANSHIPMENT



PDPTW WITH TRANSHIPMENT

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- Each node must be visited by a single vehicle only once.
- Each vehicle can pick up or deliver more than one supplier or customer.
- Pickup and delivery routes start and end at the cross-dock.
- Loads to pickup/deliver at problem nodes are known data.
- The total amounts unloaded at the receiving dock and loaded in the shipping dock should be equal. There is no end inventory at the cross-dock facility.
- Service time windows for the nodes are usually specified.
- The problem goal is to minimize the total transportation cost while satisfying all customer requests.

VEHICLE ROUTING AND SCHEDULING PROBLEMS

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VRPCD-SCM	Vehicle routing and scheduling problem with crossdocking in a multi-echelon supply chain

- Determine the best short-term operational planning of multi-echelon transportation networks comprising factories, warehouses, and customers.

- Handle different types of distribution strategies like direct shipping, shipping via DC or regional warehouses, and hybrid networks.

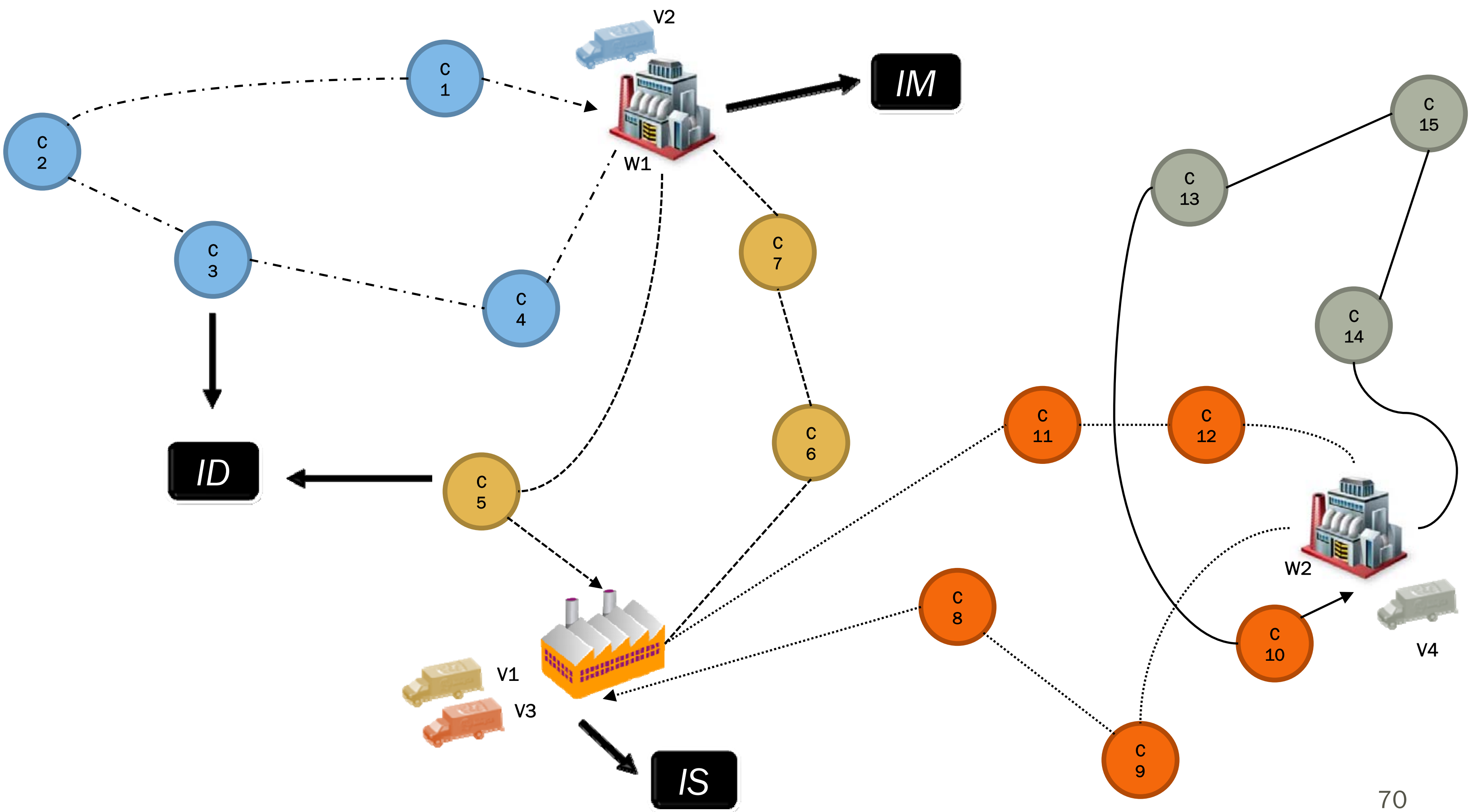
- Resemble the logistics activities at multi-site manufacturing firms by allowing multiple events at every location.

- **Types of nodes**
  - *"Pure" source nodes (IS),* usually manufacturer storages, where vehicles carry out pickup operations
  - *Mixed nodes (IM)*, like DCs, where visiting vehicles can accomplish pickup and/or unloading operations.
  - *Destination nodes (ID)*, like consumer zones, where visiting trucks just perform delivery operations

- **Number of events.** The proposed number of events for a location must be at least equal to the optimal number of vehicles stops at that node to accomplish pickup/delivery operations.

- **Global precedence.** For each vehicle stop(n,i), the model provides all the visits the vehicle has made before.

VRP-SCM


V2
C 1
C 2
W1
IM
C 13
C 15
C 3
C 4
C 7
C 14
ID
C 5
C 6
C 11
C 12
W2
C 8
C 10
V4
V1
V3
IS
C 9

MODEL VARIABLES IN VRPTW-SCM

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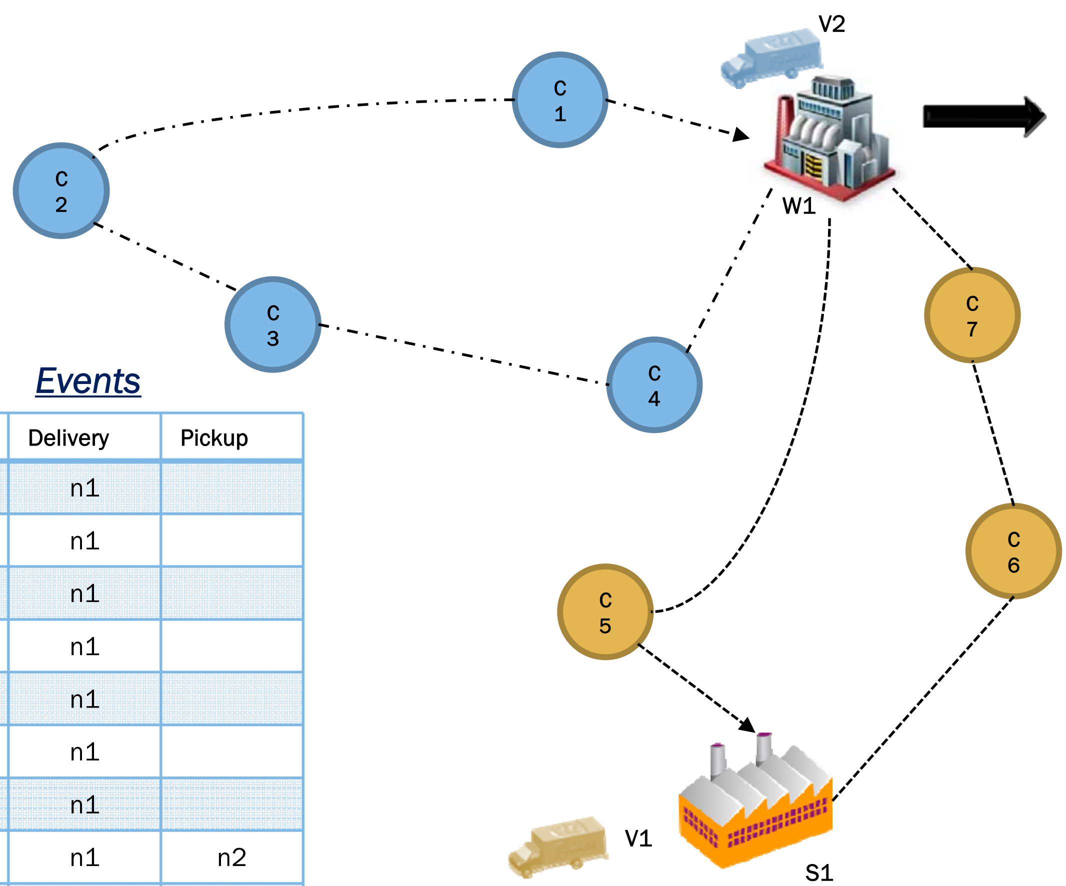
- Binary variables
  - ***Assignment variable*** Y_{niv}: the event *n* at location *i* has been allocated to vehicle *v* whenever $Y_{niv} = 1$.
  - ***Sequencing variable*** $X_{ni,n'i'}$: the vehicle stop (*n,i*) at node *i* will occur earlier than the event *n'* at site *i'* whenever $X_{ni,n'i'} = 1$ and both nodes (*i,i'*) nodes are visited by the same vehicle.

- Continuous variables
  - ***Cost-variable*** C_{ni}: distance-based transportation cost incurred by the visiting vehicle to move from the base up to stop (*n,i*) along the assigned route.
  - ***Time-variable*** T_{ni}: total time required by the assigned vehicle to travel from the base up to stop (*n,i*).
  - ***Pickup-variable*** $L_{ni,pv}$: amount of product *p* picked up by vehicle *v* during stop (*n,i*)
  - ***Delivery-variable*** $U_{ni,pv}$: amount of product *p* delivered by vehicle v to location *i* at event *n.*
  - ***Accumulated-variables*** $AL_{ni,pv}$ / $AU_{ni,pv}$: accumulated amount of product *p* picked up/delivered by vehicle *v* along the assigned route from the base to stop (*n,i*),

VRP-SCM



$$Y_{n_1 w_1 v_1} = 1$$

$$Y_{n_2 w_1 v_2} = 1$$

$$Y_{n_1 c_1 v_2} = 1$$

$$X_{n_2 w_1, n_1 c_1} = 1$$

Events

Node	Delivery	Pickup
C1	n1	
C2	n1	
C3	n1	
C4	n1	
C5	n1	
C6	n1	
C7	n1	
W1	n1	n2
S1		n1

VRP-SCM FEATURES

- Multiple products are distributed from manufacturing plants and warehouses to customers.
- Customer requests may involve several products and do not have predefined suppliers
- The amounts of products to be picked-up by a vehicle at source nodes are not predefined.
- Multiple partial shipments to a customer location are allowed
- Milk runs are performed on both sides, i.e. inbound and outbound sides.

VRP-SCM FEATURES

- Problem events are the vehicle stops at DCs and customer locations.
- Pre-specified events for each site are timely ordered and its number is chosen by the user.
- Several events can sequentially occur at any site so that a location can be visited either several times by the same vehicle or by multiple trucks.
- A vehicle can accomplish pickup and delivery operations during a stop at DCs.
- The magnitude and composition of the cargo transported by a vehicle at any stop must be traced in order to meet:
  - *capacity constraints at pickup locations*
  - *product availability constraints at delivery points*

VRP-SCM FEATURES

- Each vehicle must finally return to the assigned depot (tour constraint).
- Customer time windows and the specified maximum service time must be respected.
- Finite inventories at manufacturer storage and distribution centers at the initial time are known.
- In addition to customer demands, specific replenishment orders from DCs are to be fulfilled.
- Cross-docking at DCs is not permitted. Customer demands should be satisfied using the initial inventories.
- A DC may be predefined as the supplier of products to some customers close to this facility.

VRP-SCM FEATURES

- The problem goal is to minimize the total transportation cost while satisfying the customer service-level requirements.

- Transportation costs include:
  - fixed expenses incurred by used vehicles,
  - distance-based variable costs, mainly fuel costs
  - time-based variable costs, mainly driver wages.

VRPTW-SCM MODEL CONSTRAINTS

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- **Route building constraints** assigning a particular stop (*n*,*i*) to at most a single vehicle, and ordering vehicle stops (*n*,*i*) on the same route.

- **Product inventory constraints** restraining the overall amount of products loaded by visiting vehicles at source nodes accounting for product availability.

- **Additional inventory constraints** monitoring the amount of every product received at each warehouse over the planning horizon.

- **Product demand constraints** ensuring that customer requests are satisfied.

VRPTW-SCM MODEL CONSTRAINTS

- **Null in-transit inventory constraints** requiring that every product unit picked up by a vehicle must be delivered to a customer before the end of the vehicle trip.

- **Loading/unloading constraints** monitoring the total amount of products transported by each vehicle to prevent from overcapacity or product shortages.

- **Time window and maximum service time constraints** ensuring that the customer service begins within the specified TW, and each vehicle returns to its base within the allowed working period.

Vehicle routing constraints in SCM

Allocating vehicles to depots

$$\sum_{l \in B_v} W_{lv} \leq 1 \quad \forall v \in V$$

Assigning the event n at node i to vehicle v

$$\sum_{v \in V_i} Y_{nv} \leq 1 \quad \forall n \in N_i, i \in I$$

Preordering of time events predefined for node i

$$\sum_{v \in V_i} Y_{nv} \geq \sum_{v \in V_i} Y_{n'v} \quad \forall (n, n') \in N_i, i \in I : n < n'$$

Activated vehicle condition

$$\sum_{i \in I_v} \sum_{n \in N_i} Y_{nv} \leq M \sum_{l \in B_v} W_{lv} \quad \forall v \in V$$

Travel cost and time from the vehicle depot to the first visited node

$$\left\{ \begin{array}{l} C_n \geq \sum_{l \in B_v} dc_v \, d_{li} \, W_{lv} - M_C (1 - Y_{nv}) \\ T_n \geq \sum_{l \in B_v} \left(\frac{d_{li}}{sp_v}\right) W_{lv} - M_T (1 - Y_{nv}) \end{array} \right\} \quad \forall n \in N_i, i \in I, v \in V_i$$

Travel cost and time from the base to vehicle stop (i,n)

$$\left\{ \begin{array}{l} C_{n'} \geq C_n + dc_v \, d_{ii'} - M^C (1 - X_{nn'}) - M^C (2 - Y_{nv} - Y_{n'v}) \\ C_n \geq C_{n'} + dc_v \, d_{i'i} - M^C X_{nn'} - M^C (2 - Y_{nv} - Y_{n'v}) \\ T_{n'} \geq T_n + ft_i + \sum_{p \in P_i} vt_{ip} (L_{npv} + U_{npv}) + \frac{d_{ii'}}{sp_v} - M^T (1 - X_{nn'}) - M^T (2 - Y_{nv} - Y_{n'v}) \\ T_n \geq T_{n'} + ft_{i'} + \sum_{p \in P_{i'}} vt_{i'p} (L_{n'pv} + U_{n'pv}) + \frac{d_{i'i}}{sp_v} - M^T X_{nn'} - M^T (2 - Y_{nv} - Y_{n'v}) \end{array} \right\}$$

$$\forall n \in N_i, n' \in N_{i'}, i, i' \in I, v \in V_{ii'} : i < i'$$

Bound on the routing cost and time for the tour assigned to vehicle v

$$\left\{ \begin{array}{l} CV_v \geq C_n + \sum_{l \in B_v} dc_v \, d_{il} \, W_{lv} - M_C (1 - Y_{nv}) \\ TV_v \geq T_n + ft_i + \sum_{p \in P_i} vt_{ip} (L_{npv} + U_{npv}) + \sum_{l \in B_v} \frac{d_{il}}{sp_v} W_{lv} - M_T (1 - Y_{nv}) \end{array} \right\}$$

$$\forall n \in N_i, i \in I, v \in V_i$$

Time-window and maximum service time constraints

$$a_i \leq T_n \leq b_i \quad n \in N_i, i \in ID$$

$$TV_v \leq t_v^{\max} \quad v \in V$$

VRP-SCM FORMULATION

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Vehicle cargo constraints in SCM

Product availability constraints

$$\sum_{v \in V_i} \sum_{n \in N_i} L_{npv} \le INV_{ip} \qquad \forall i \in (IS_p \cap IM_p), p \in P$$

Product demand constraints

$$\sum_{v \in V_i} \sum_{n \in N_i} U_{npv} = DEM_{ip} - BL_{ip} \qquad \forall i \in (ID_p \cap IM_p), p \in P$$

Null in-transit inventory constraints

$$\sum_{n \in N_i} \sum_{i \in IS \cup IM} L_{npv} = \sum_{n \in N_i} \sum_{i \in IM \cup ID} U_{npv} \qquad , p \in P, v \in V$$

Vehicle loading/unloading operation constraints

$$\begin{Bmatrix} L_{npv} \le INV_{ip} Y_{nv} \\ U_{npv} \le DEM_{ip} Y_{nv} \end{Bmatrix} \quad \begin{matrix} \forall n \in N_i, i \in (IS_p \cap IM_p), p \in P, v \in V_i \\ \forall n \in N_i, i \in (ID_p \cap IM_p), p \in P, v \in V_i \end{matrix}$$

Accumulated amount of product p picked-up by vehicle v up to the stop (i,n)

$$\begin{Bmatrix} AL_{n'pv} \ge AL_{npv} + L_{n'pv} - M_L(1 - X_{nn'}) - M_L(2 - Y_{nv} - Y_{n'v}) \\ AL_{npv} \ge AL_{n'pv} + L_{npv} - M_L X_{nn'} - M_L(2 - Y_{nv} - Y_{n'v}) \end{Bmatrix}$$

$$\forall n \in N_i, n' \in N_{i'}, i, i' \in I, p \in P_{ii'}, v \in V_{ii'} (i,n) < (i', n')$$

Accumulated amount of product p delivered by vehicle v up to the stop (i,n)

$$\begin{Bmatrix} AU_{n'pv} \ge AU_{npv} + U_{n'pv} - M_L(1 - X_{nn'}) - M_L(2 - Y_{nv} - Y_{n'v}) \\ AU_{npv} \ge AU_{n'pv} + U_{npv} - M_L X_{nn'} - M_L(2 - Y_{nv} - Y_{n'v}) \end{Bmatrix}$$

$$\forall n \in N_i, n' \in N_{i'}, i, i' \in I, p \in P_{ii'}, v \in V_{ii'} : (i,n) < (i', n')$$

Maximum volumetric and weight vehicle capacity constraints

$$\begin{Bmatrix} \sum_{p \in P} uv_p (AL_{npv} - AU_{npv}) \le vq_v Y_{nv} \\ \sum_{p \in P} uw_p (AL_{npv} - AU_{npv}) \le wq_v Y_{nv} \end{Bmatrix} \qquad \forall n \in N_i, i \in I, p \in P, v \in V_i$$

Lower bounds on the cargo transported by vehicle v after stop (i,n)

$$\begin{Bmatrix} AL_{npv} \ge AU_{npv} \\ AL_{npv} \ge L_{npv} \\ AU_{npv} \ge U_{npv} \end{Bmatrix} \qquad \forall n \in N_i, i \in I, p \in P, v \in V_i$$

Upper bounds on the accumulated amount of product p loaded/unloaded by vehicle v after stop (i,n)

$$\begin{Bmatrix} AL_{npv} \le \sum_{i' \in I} \sum_{n' \in N_{i'}} L_{n'pv} \\ AU_{npv} \le \sum_{i' \in I} \sum_{n' \in N_{i'}} U_{n'pv} \end{Bmatrix} \qquad \forall n \in N_i, i \in I, p \in P, v \in V_i$$

VRP-SCM FORMULATION

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Objective Function

The sum of distance-based travel costs and vehicle fixed costs

$$Min \quad \sum_{v \in V} CV_v + \sum_{v \in V} \sum_{l \in B_v} fc_v \, W_{lv}$$

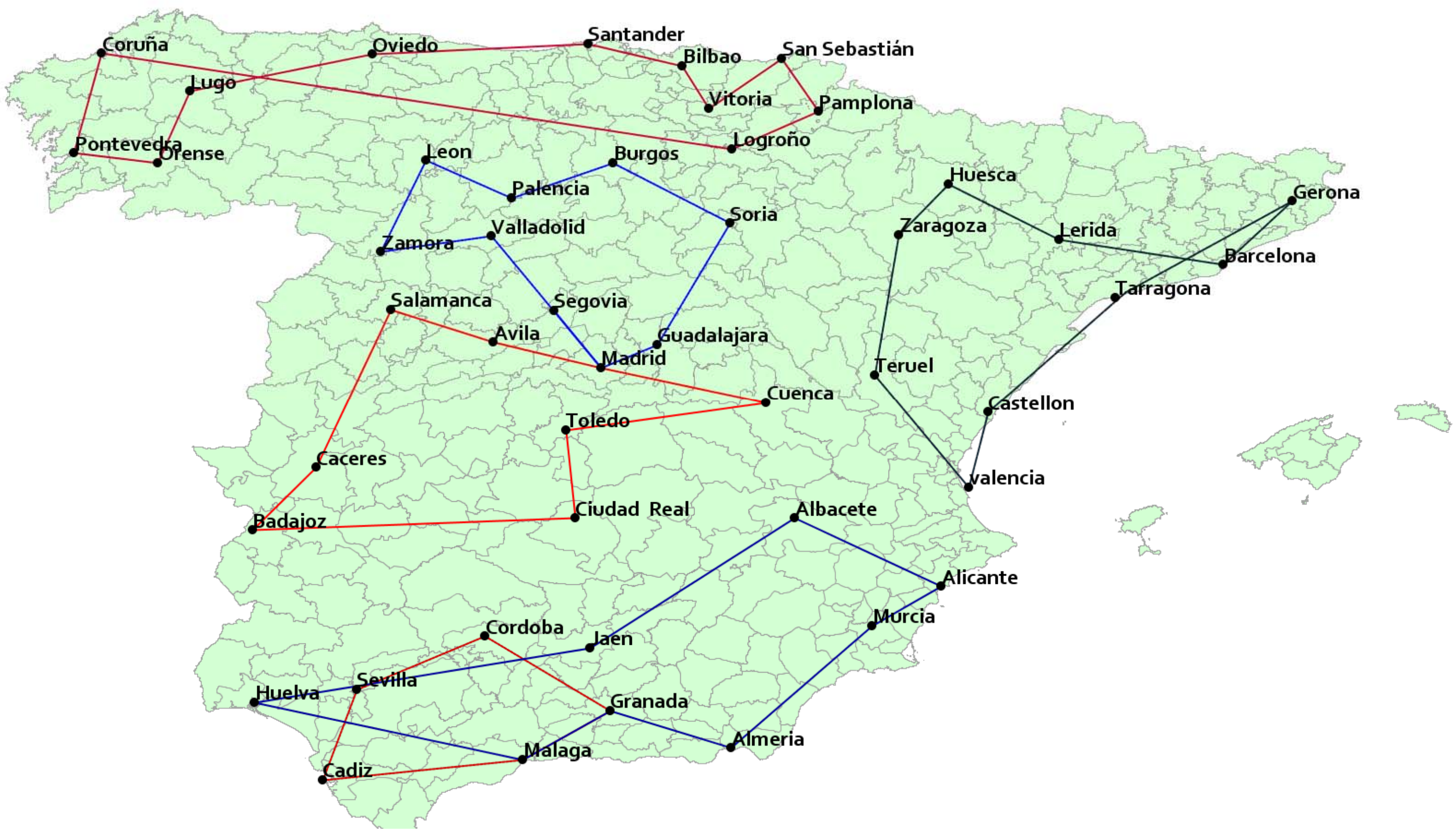
The weighted sum of distance-based and time-based travel costs plus vehicle fixed costs

$$Min \quad \sum_{v \in V} CV_v + \sum_{v \in V} \sum_{l \in B_v} fc_v \, W_{lv} + \sum_{v \in V} utc_v \, TV_v$$

Fixed and variable transportation costs plus the penalties for unsatisfied demands, late services and overtime journeys

$$Min \quad \sum_{v \in V} CV_v + \sum_{v \in V} \sum_{l \in B_v} fc_v \, W_{lv} + \sum_{v \in V} utc_v \, TV_v + \sum_{v \in V} (co_v \, OVT_v + \sum_{i \in ID_v} \sum_{n \in N_i} cl_i \, TD_n) + \sum_{p \in P} \sum_{i \in I} c_{B,i} \, B_{ip}$$

VRP-SCM SOLUTION



VEHICLE ROUTING AND SCHEDULING PROBLEMS

TSP	Travelling salesman problem
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VRPCD-SCM FEATURES

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- **Generalization of the VRP-SCM problem** to consider the possibility of cross-docking.
- **Intermediate depots may keep finite stocks** of fast-moving products (warehousing) and act as cross-dock platforms for slow-moving, high-value items.
- **Replenishment orders and cross-docking operations** are triggered **when the initial stock in a warehouse is insufficient** to meet the demand of the assigned customers.
- Inbound and outbound vehicles must stay in receiving/shipping docks of DCs until they complete their delivery/pickup tasks.
- Target product inventories at the end of the planning horizon may be considered.

VRPCD-SCM FEATURES

- **Product inventories at cross-dock facilities** must be traced over the planning horizon.

- The problem goal aims to **minimize fixed and variable transportation costs.**

VRPCD-SCM MILP MODEL

Route building constraints

Allocating base nodes to vehicles

$$\sum_{l \in IB_v} \sum_{n \in N_l} Y_{nlv} \leq 1 \qquad v \in V$$

Allocating events at every node to vehicles

$$\sum_{v \in V_i} Y_{niv} \leq 1 \qquad n \in N_i, i \in I$$

Pre-ordering events occurring at the same node

$$\sum_{v \in V_i} Y_{niv} \geq \sum_{v \in V_i} Y_{n'iv} \quad (n,n') \in N_i : n' > n, i \in I$$

Used vehicle condition

$$\sum_{i \in I_v} \sum_{n \in N_i} Y_{niv} \leq M_v \left(\sum_{l \in IB_v} \sum_{n \in N_l} Y_{nlv} \right) \qquad v \in V$$

Travelling cost constraints

Travelling cost from the base node l to the first serviced node i for vehicle

$$C_{ni} \geq \sum_{l \in IB_v} \sum_{n' \in N_l} c_{li} \, Y_{n'lv} - M_C \left(1 - Y_{niv}\right) \qquad n \in N_i, i \in I, v \in V_i$$

Accumulated travelling cost for vehicle v up to the stop (n,i)

$$\begin{Bmatrix} C_{n'i'} \geq C_{ni} + c_{ii'} - M_C \left(1 - X_{ni,n'i'}\right) - M_C \left(2 - Y_{niv} - Y_{n'i'v}\right) \\ C_{ni} \geq C_{n'i'} + c_{i'i} - M_C \; X_{ni,n'i'} - M_C \left(2 - Y_{niv} - Y_{n'i'v}\right) \end{Bmatrix}$$

$$n \in N_i, n' \in N_{i'}, (i,i') \in I, v \in V_{ii'} : i < i'$$

Overall travelling cost for vehicle v

$$CV_v \geq C_{ni} + \sum_{l \in B_v} \sum_{n' \in N_l} c_{li} \, Y_{n'lv} - M_C \left(1 - Y_{niv}\right)$$

$$n \in N_i, i \in I, v \in V$$

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Travelling time constraints

Travelling cost from the assigned base node $l \in IB_v$ to the first serviced node for vehicle v

$$T_{ni} \geq \sum_{l \in IB_v} \sum_{n' \in N_l} t_{li} \, Y_{n'lv} + ft_l + vt_l \sum_{p \in P_l} L_{nlpv} - M_C \left(1 - Y_{niv}\right)$$
$$n \in N_i, i \in I, v \in V$$

Travelling time for vehicle v from the assigned base node to the stop (n,i)

$$\left\{ \begin{array}{l} T_{n'i'} \geq T_{ni} + ft_i + vt_i \left(\sum_{p \in P_i} L_{ni,pv} + U_{ni,pv} \right) + t_{ii'} - M_C \left(1 - S_{ni,n'i'}\right) - M_C \left(2 - Y_{niv} - Y_{n'i'v}\right) \\ T_{n'i'} \geq T_{ni} + ft_{i'} + vt_{i'} \left(\sum_{p \in P_i} L_{n'i',pv} + U_{n'i',pv} \right) + t_{i'i} - M_C S_{ni,n'i'} - M_C \left(2 - Y_{niv} - Y_{n'i'v}\right) \end{array} \right\}$$
$$n \in N_i, n' \in N_{i'}, (i,i') \in I, v \in V_{ii'} : i < i'$$

Overall travelling time for vehicle v

$$OT_v \geq T_{ni} + ft_i + vt_i \left(\sum_{p \in P_i} L_{ni,pv} + U_{ni,pv} \right) + \sum_{l \in B_v} \sum_{n' \in N_l} t_{il} Y_{n'lv} - M_C \left(1 - Y_{niv}\right) \qquad n \in N_i, i \in I, v \in V$$

Time window and maximum service time constraints

$$a_i \leq T_{ni} \leq b_i \qquad n \in N_i, i \in I$$

$$OT_v \leq t_v^{\max} \qquad v \in V$$

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Vehicle-related constraints

Overall product balance for every vehicle

$$\sum_{i \in IS \cup IM} \sum_{n \in N_i} L_{ni,pv} = \sum_{i \in (IM \cup ID)} \sum_{n \in N_i} U_{ni,pv} \qquad p \in P, v \in V$$

Accumulated amount of product p picked up by vehicle v up to the stop (n,i)

$$\begin{Bmatrix} AL_{n'i',pv} \geq AL_{ni,pv} + L_{n'i',pv} - M_L\left(1 - S_{ni,n'i'}\right) - M_L\left(2 - Y_{niv} - Y_{n'i'v}\right) \\ AL_{ni,pv} \geq AL_{n'i',pv} + L_{ni,pv} - M_L\, S_{ni,n'i'} - M_L\left(2 - Y_{niv} - Y_{n'i'v}\right) \end{Bmatrix} \quad n \in N_i, n' \in N'_{i'}, (i,i') \in I, p \in P, v \in V_{ii'} : n < n', i \neq i'$$

$$AL_{n'i,pv} \geq AL_{ni,pv} + L_{n'i,pv} - M_L\left(2 - Y_{niv} - Y_{n'iv}\right) \qquad (n,n') \in N_i, i \in I, v \in V_i : n < n'$$

Accumulated amount of product p delivered by vehicle v up to the stop (n,i)

$$\begin{Bmatrix} UL_{n'i',pv} \geq UL_{ni,pv} + U_{n'i',pv} - M_L\left(1 - S_{ni,n'i'}\right) - M_L\left(2 - Y_{niv} - Y_{n'i'v}\right) \\ UL_{ni,pv} \geq UL_{n'i',pv} + U_{ni,pv} - M_L\, S_{ni,n'i'} - M_L\left(2 - Y_{niv} - Y_{n'i'v}\right) \end{Bmatrix} \quad n \in N_i, n' \in N'_i, (i,i') \in I, p \in P, v \in V_{ii'} : n < n', i \neq i'$$

Vehicle capacity constraints

$$\begin{Bmatrix} \sum_{p \in P} uw_p\left(AL_{ni,pv} - AU_{ni,pv}\right) \leq qw_v \\ \sum_{p \in P} uv_p\left(AL_{ni,pv} - AU_{ni,pv}\right) \leq qv_v \\ AL_{ni,pv} - AU_{ni,pv} \geq 0 \end{Bmatrix} \quad n \in N_i, i \in I_v, v \in V$$

Bounds on variables $AU_{ni,pv}$ and $AL_{ni,pv}$

$$\begin{Bmatrix} L_{ni,pv} \leq AL_{ni,pv} \leq \sum_{i' \in IS \cup IM} \sum_{n' \in N_i'} L_{n'i',pv} \\ U_{ni,pv} \leq AU_{ni,pv} \leq \sum_{i' \in IS \cup IM} \sum_{n' \in N_i'} U_{n'i',pv} \end{Bmatrix} n \in N_i, i \in I_v, v \in V, p \in P$$

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Additional inventory received at cross-docking facilities from other sources

Additional inventory received at cross-docking facilities from other sources

$$AI_{n'ip} \geq AI_{nip} + \sum_{v \in V_i} U_{n'i,pv} \qquad (n,n') \in N_i, i \in IM, p \in P : n < n'$$

Bounds for the value of AI_{nip}

$$\sum_{v \in V_i} U_{ni,pv} \leq AI_{nip} \leq \sum_{n' \in N_i} \sum_{v \in V_i} U_{n'i,pv} \qquad n \in N_i, i \in IM, p \in P$$

Objective Function

$$Min \left[\sum_{v \in V} CV_v + \sum_{v \in V} \sum_{l \in IB_v} \sum_{n \in N_l} fc_v \, Y_{nlv} \right]$$

$$Min \left[\sum_{v \in V} OT_v \right]$$

CONCLUSIONS

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- Transportation is a **significant link between different stages** in a global supply chain.
- Small reductions in transportation expenses could result in **substantial total savings** over a number of years.
- The use of **vehicle routing and scheduling models and techniques** can be instrumental in realizing those savings.
- Different types of vehicle routing problems have been studied over the years; most of them dealing **with single-echelon networks and a single type of operation** (pickup or delivery) at every location.
- Since they are NP-hard, solution methods based on **metaheuristic techniques** are generally applied.
- Recently, **new model-based approaches** have been developed for the operational planning of **multi-echelon distribution networks**
- The **so-called VRPCD-SCM** problem includes many features usually arising in the operation **of real-world distribution networks**.
- Further work on this area is still under way in order to **address current industrial needs**

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THANKS FOR YOUR ATTENTION

QUESTIONS ?